

# NRF **심층** 정책 REPORT

## 양자 컴퓨팅 기술 동향·실체(이론·모사)의 이해와 "기술 초격차 확장 시대" 도래의 인식 필요성

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# 표 및 그림 목차

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# I.

“ ” ) 가 가 . ( (Quantum Mechanics) “ (Phono Sapiens)” 가 가 , , 가 , 가 가 . ”가 . “ ” ; 가 가 . ; 가 . NASA Cold Atom Lab. 4 ( ) (Cold Atom Lab: Celebrating Four Years of Quantum Science in Space) 가 , ( , ). , 가 가 . , , .

VI

IV

, ( 가 )  
 가  
 (Hydrogen Atom) ( 가  
 )  
 ;  
 Mathematica Symbolic  
 , IBM Qiskit  
 PyTorch  
 가 가 ,  
 , 18  
 가 , “ ”  
 (Langlands Program), 가 (Prime Number),  
 가 (Discrete), 가  
 (Non-commutative)  
 ,  
 가  
 가 가 가  
 (Product)  
 ,  
 [38] [39]  
 ( , , )

## II. 가

1

18 (Johann Carl Friedrich Gauß IPA, 30 April 1777 – 23 February 1855) 가 (Fundamental Theorem of Algebra, FTA) .

( “FTA” ): “ .” . ( [ 1] , ) . FTA (Root) “ ” “ ” , FTA “ ” “ ” “ ” 가 가 . 가 가 .

**대수학의 기본 정리**(代數學의 基本 定理 ; fundamental theorem of algebra)란 상수가 아닌 복소계수 다항식은 적어도 하나의 영점을 갖는다는 정리이다.

즉, 복소계수 다항식

$$p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0, \quad a_n \neq 0, \quad n \geq 1$$

에 대해  $p(a) = 0$  인 복소수  $a$  가 적어도 하나는 존재한다는 것이다.

이 정리는 복소수체가 실수체와는 달리 대수적으로 닫힌 체임을 뜻한다.

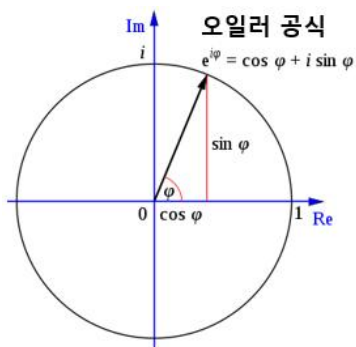
**Theorem.** *Every linear operator on a finite-dimensional complex vector space has an eigenvalue.*

FTA Ref.: Wikipedia

[ 1] 가

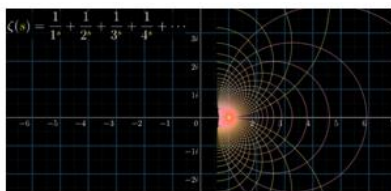
$x^2 - 1 = 0$  가  $x = \pm 1$  .  $x^2 - 3x + 2 = 0$  가  $x = 1, x = 2$   
 . ,  $ax^2 + bx + c = 0$  가 ?  
 가 . ,  $x^2 + 1 = 0$  가 . 가?  
 0 1, , , 가 가 , “ 가 ”  $e^{\pi i} + 1 = 0$   
 (Leonhard Euler, 15 April 1707 - 18 September  
 1783) ,  $x^2 + 1 = 0$   $x = \pm i$  ( $i := \sqrt{-1}$ )  
 . “ ” ,  $x = \pm i$  . “ ”  
 “ ” ? 18 . “ ”  
 .  
 “ ” “ ” . “ ”  
 , “ ” .  $\mathbb{N}$   
 $\mathbb{N} := \{1, 2, 3, \dots\}$  ,  $\mathbb{N}$  (+) “ ” . “ ”  
 ;  $\mathbb{N}$   $\mathbb{N}$  . (-) “ ”  
 . 가  $\mathbb{N}$  ,  $\mathbb{N}$  “ ”  
 .  $\mathbb{Z} := \{\dots, -2, -1, 0, 1, 2, \dots\}$  “ ” (-)  
 “ ” .  
 “ ” .  
 (Algebraically Closed) .  
 .  $\mathbb{N}$  .  
 가 . ,  
 $\mathbb{R}$  “ ” . ,  $ax^2 + bx + c = 0$   
 “ ” ;  $i$   
 $\mathbb{C} := \{s = \sigma + i\tau : \sigma \in \mathbb{R}, \tau \in \mathbb{R}\}$  FTA “ ” “ ” .  
 ,  $a, b, c, x$ 가 가  $ax^2 + bx + c = 0$  .  
 가? 가  $e^{it} = \cos t + i \sin t$  가  
 .  $e^{it} := \exp(it)$   
 .  
 FTA , “ ” “ ” “ ” . 가

,  $\mathbb{C}$  “ ” “ ”  $\mathbb{C}$   
 (Complex Number Field) (Algebraically Closed)  
 가 ;  $(-)$   $\mathbb{N}$   $\mathbb{Z}$  ,  
 $\mathbb{R}$   $\mathbb{C}$  가 .  
 18 가 .  
 .  $\mathbb{C}$   
 가 .  $\mathbb{C}$   
 . ;  
 . ( ,  
 가  $\left(\frac{\sin \pi u}{\pi u}\right)^2$   
 .)



$$e^{i\pi} + 1 = 0$$

해석적 연속이 필요한 물리 이론(Physics Theory) 다수 존재

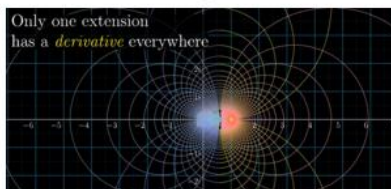


$$1 + 2 + 3 + 4 + \dots = \infty$$

$$1 + 2 + 3 + 4 + \dots = -\frac{1}{12}$$

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$$

$$\zeta(-1) = -\frac{1}{12}$$



해석적 연속 전의  $\zeta(-1) = \infty$   
 해석적 연속 후의  $\zeta(-1) = -1/12$   
 (Some intrinsic connection?)

“해석적 연속은 유일성을 가짐”

해석적 연속의 결과는  
 그 표현의 형태와 관계 없이  
 그 결과가 항상 유일함.

Figures Ref.: Wikipedia, namu.wiki  
 [ 2] , ,

[ 2] ( “ ” ) .  
 ( $\infty$ )  $\mathbb{N}$  “ ” . “  
 ” “ 가”

가 0 “ ” ; [ 1]. , C 가  
 가 “∞” . “ ”  
 $\lim_{x \rightarrow 1}$   $\lim_{x \rightarrow \infty}$  가 가 ∞  
 “ ” (Number) 가  
 , 가 .  
 C , ∞  
 C ( [ 1])  
 [ 2] .  
 [1].

가 (Relativistic Quantum Field Theory)  
 (Casimir Effect) , (Photon) 가 0 1 (L)  
 (Zero-point Energy)가 , [ 1].

$$\mathcal{E} = \frac{\pi \hbar c}{L} \sum_{p=1}^{\infty} p = \frac{\pi \hbar c}{L} (1+2+\dots) = \frac{\pi \hbar c}{L} \left( -\frac{1}{12} \right)$$

C 가 [ 2].  
 가 , (Prime Number)  
 ( “ ” ). (Robert Phelan  
 Langlands, 6 October 1936 - ) (Langlands Program)[2][ 2]  
 “  
 ” ( ),

$$K = \mathbb{Q}(\alpha) = \left\{ \sum_{n=0}^m a_n \alpha^n : a_n \in \mathbb{Q} \right\}$$

( $K$ : 유리수체  $\mathbb{Q}$ 의 대수적 유한 확대체,  $\alpha$ : 유리수 계수를 갖는 다항함수의 근)

$$\mathbb{Q} \subset \mathbb{Q}(\alpha_1) \subset \mathbb{Q}(\alpha_2) \subset \dots \subset \mathbb{Q}(\alpha_n) \subset \dots \subset \mathbb{C}$$

1) (Automorphic Form) ( “ ” )

2) (Prime)

3) (Zero )  $\alpha$

4)  $\mathbb{C}$

( “ ”가 )  $f(az) = bf(z)$

[ 1];

(Discrete) (Continuous)

$\mathbb{C}$ 가

$\mathbb{C}$ 가

;

$\sqrt{-23}$

$f(x) = x^3 + ax + b$

$\mathbb{Q} \cup \{ \sqrt{-23} \}$

“ ”  $\mathbb{Q}(\sqrt{-23})$

$0.5 + 3\sqrt{-23} \in \mathbb{Q}(\sqrt{-23})$

$p(x) = ax^2 + bx + c$

$Disc(p) = b^2 - 4ac$

$Disc(f) = -4a^3 - 27b^2$

$a = -1, b = \pm 1$

$Disc(f) = 23$

$f(x) = x^3 + x \pm 1$

$$f(q) = \sum_{n=1}^{\infty} a_n q^n = q \prod_{n=1}^{\infty} (1 - q^n)(1 - q^{23n})$$

$p \ (p \neq 23) \in \mathbb{Q}(\sqrt{-23})$ ,  $a_p = 2$   
 $\langle 1 \rangle$

(Approximation)

$$f(q) = q^1 - q^2 - q^3 + \dots + q^{58} + 2q^{59} + \dots + 2q^{101} + \dots + 2q^{167} + \dots + 2q^{173} + \dots + 2q^{211} + \dots + 2q^{223} + \dots + 2q^{271} + \dots$$

$$a_n = 2 \quad n \in \mathbb{N} \quad \langle 1 \rangle \quad (a_p = 2)$$

(Polynomial Time)  $a_p$   
 [3].

**Theorem** (Edixhoven et al.). *Let  $p$  be a prime. There is an algorithm to compute  $\tau(p)$ , for prime  $p$ , that is polynomial-time in  $\log(p)$ . More generally, if  $f = \sum a_n q^n$  is a Hecke eigenform in some space  $M_k(\Gamma_1(N))$ , where  $k \geq 2$ , then there is an algorithm to compute  $a_p$  in time polynomial in  $\log(p)$ .*

$\langle 1 \rangle \ a_n = 2 \quad n \in \mathbb{N} \quad i \in \mathbb{N} \quad p_i \in \mathbb{Q}(\sqrt{-23}) \quad \mathbb{Z}[\sqrt{-23}]$   
 $p_i = (x + y\sqrt{-23})/d \quad (n, i) \quad , \ n \leq 300$

$n$	$i$	$(x^2 + 23) \bmod n \equiv 0$ 을 만족하는 $x \in \mathbb{Q}(\sqrt{-23})$ ‘ ’			
		$x$	$p_i = (x + y\sqrt{-23})/d$	$x$	$p_i = (x + y\sqrt{-23})/d$
59	17	6	$59 = (6 - \sqrt{-23})(6 + \sqrt{-23})$	53	$59 = (53 - \sqrt{-23})(53 + \sqrt{-23})/48$
101	26	49	$101 = (49 - \sqrt{-23})(49 + \sqrt{-23})/24$	52	$101 = (52 - \sqrt{-23})(52 + \sqrt{-23})/27$
167	39	12	$167 = (12 - \sqrt{-23})(12 + \sqrt{-23})$	155	$167 = (155 - \sqrt{-23})(155 + \sqrt{-23})/144$
173	40	82	$173 = (82 - \sqrt{-23})(82 + \sqrt{-23})/39$	91	$173 = (91 - \sqrt{-23})(91 + \sqrt{-23})/48$
211	47	71	$211 = (71 - \sqrt{-23})(71 + \sqrt{-23})/24$	140	$211 = (140 - \sqrt{-23})(140 + \sqrt{-23})/93$
223	48	73	$223 = (73 - \sqrt{-23})(73 + \sqrt{-23})/24$	150	$223 = (150 - \sqrt{-23})(150 + \sqrt{-23})/101$
271	58	93	$271 = (93 - \sqrt{-23})(93 + \sqrt{-23})/32$	178	$271 = (178 - \sqrt{-23})(178 + \sqrt{-23})/117$

가  
 가 [ 2]. ,  
 “ 가 ”  
 (Electron) (Spin)  
 (  $x : \uparrow, y : \downarrow$  )

$$\begin{cases} 0x + 1y = 1u \\ 1x + 0y = 1v \end{cases} \Rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow \begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} \text{ 해가 무수히 많을 조건: } \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

무수히 많은 해 :  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ +1 \end{pmatrix} = -1 \begin{pmatrix} -1 \\ +1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ +2 \end{pmatrix} = -1 \begin{pmatrix} -2 \\ +2 \end{pmatrix}, \dots, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -t \\ +t \end{pmatrix} = E \begin{pmatrix} -t \\ +t \end{pmatrix}, \dots$

무수히 해가 많으려면  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = E \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & E \\ E & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} -E & 1 \\ 1 & -E \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

무수히 해가 많으려면  $\begin{pmatrix} -E & 1 \\ 1 & -E \end{pmatrix}$ 의 역행렬이 없어야 한다(있다면, 양변에 역행렬 곱해져  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ 로 결정).

위에서 역행렬이 없다  $\Leftrightarrow$  행렬식이 0이다:  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = E \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} a-E & b \\ c & d-E \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ 에서  $(a-E)(d-E) - bc = 0$ .

대수학의 기본 정리에 따라,  $f(E) = (a-E)(d-E) - bc = 0$ 의 해(복소수)가 항상 존재한다( $a, b, c, d$ 가 복소수일 때).

양자물리학은 계수  $H = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 인 연립방정식의 해가 무수히 많아지기 위한 쌍,  $\begin{pmatrix} x \\ y \end{pmatrix}, E$ 들을 찾는다(주로  $E$ 의 허수부가 0).

### H가 가

$H$  “ 가 ” “ 가”가  $E$   
 “ 가 ” 가 ( “ ”가 )  
 가

무수히 해가 많으려면  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = E \begin{pmatrix} x \\ y \end{pmatrix}$ 에서  $(0-E)(0-E) - 1 \times 1 = 0$  이어야 한다.

무수히 해가 많으려면  $E^2 = 1$  이어야하므로, 임의의 복소수  $t$ 에 대해  $\begin{pmatrix} x \\ y \end{pmatrix}, E = \begin{pmatrix} \pm 1 \\ 1 \end{pmatrix} t, \pm 1$ 이면 해가 무수히 많다.

### “ ” “ (Coherence) ”

연립방정식의 해가 무수히 많으면 방정식들이 같아진다:  $\begin{cases} 0t + 1t = +1t \\ 1t + 0t = +1t \end{cases} \Rightarrow \begin{cases} t = t \\ t = t \end{cases}$

연립방정식의 해가 무수히 많으면 방정식들이 같아진다:  $\begin{cases} 0(-t) + 1(t) = -1(t) \\ 1(-t) + 0(t) = -1(-t) \end{cases} \Rightarrow \begin{cases} t = -t \\ -t = t \end{cases} \Rightarrow \begin{cases} t = -t \\ t = -t \end{cases}$

“  $\begin{pmatrix} x \\ y \end{pmatrix}$  ” .  $H = 1$   $x$  , 2  $y$   
 , ,  $H = \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix}, E$ 가 ; ,  $E$  가 0 “ ” “ ”  
 ( 가 0  $E$  ).  
 ,  $x$   $y$ 가 ( ) (Sound Wave) ,  
 $x$   $y$ 가 “ ” “ ” , 가 0  $E$  .  
 ( ) (Standing Wave) “ ” “ ”  
 ,  $E$ 가 ;  $E$  가 0  
 “ ” “ ” - ,  
 $H$  ,  $H$   


---

 $H$   $E$   
 → 가 ,  
 가 .  
 ,  $t$  ( , ; “Span”  
 가 ). 1 ; 1  
 ;  $\sqrt{1^2+0^2} = \sqrt{1^2} = 1$ .  $x$   $y$   
 (Vector Space) .  
 $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = +1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \therefore$  고유 벡터 :  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ , 곱셈값 : +1  
 $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \therefore$  고유 벡터 :  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ , 곱셈값 : -1  
 ( ) . , 가 FTA  
 ( ) ( )  
 “ ”  
 ( ) “ ” ,  
 [ 2]. (Fermat’s Last Theorem) ,  
 < 1> [ 2].  
 (Langlands Correspondence) ,  
 ( 가 ); ,

(Algebraic Geometry)  
- 13 November 2014)

(Alexander Grothendieck, 28 March 1928  
가 [ 2].

가 , 가  
(Discrete) (Algebraic Number)가  
(Continuous) (Geometric)

가?

가

[4][5][6],

가 [ 2].

美 大

가 [39].

“ ”

(Logic) .

가

가 ,

가

가

2

(Relativistic Quantum Physics)

( ).

가 (Multiverse Theories) /M (M-Theory) (Holographic Principle) (Mirror World) [7].

(Stephen William Hawking CH CBE FRS FRSA, 8 January 1942 - 14 March 2018)가 (Hawking Radiation)

(Holographic Principle) [ 3].

2 , “ 1 ” 가 (Simulation Hypothesis) (a Simulated Universe) 가

<호킹 복사 이론> → 블랙홀의 증발 예측  
↓  
<블랙홀의 역설> → 정보손실 → 물리 이론 모순  
↓  
<해결책 1> → 홀로그램 원리 도입  
블랙홀 ← 2차원, 정보 無소실성  
(플랑크 거리 기반 단위 면적당 1비트 정보 저장)  
↓  
<홀로그램 우주 이론>  
우주가 실제로 2차원적인가,  
수학적으로 그렇게 다룰 수 있는 것 뿐인가  
↓  
<시뮬레이션 우주론, 모의실험 가설>  
우리가 살고 있는 우주는 시뮬레이션이다.

[ 3] , NRF (2022) [8]

[ 4] : 가 가 ? (Holographic Principle)

,  $3 \times 10^{65}$  .

( ; ) ; ,

가 가 ,

( $1.62 \times 10^{-35}m$ ) 1 (1993

(Gerard 't Hooft, '99 ) ).

가 “ ”

3 .

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## The Holographic Principle

f t reddit email print

Is this picture worth a thousand words? According to the Holographic Principle, the most information you can get from this image is about  $3 \times 10^{65}$  bits for a normal sized computer monitor. The Holographic Principle, yet unproven, states that there is a maximum amount of information content held by regions adjacent to any surface. Therefore, counter-intuitively, the information content inside a room depends not on the volume of the room but on the area of the bounding walls. The principle derives from the idea that the Planck length, the length scale where quantum mechanics begins to dominate classical gravity, is one side of an area that can hold only about one bit of information. The limit was first postulated by physicist Gerard 't Hooft in 1993. It can arise from generalizations from seemingly distant speculation that the information held by a black hole is determined not by its enclosed volume but by the surface area of its event horizon. The term "holographic" arises from a hologram analogy where three-dimension images are created by projecting light though a flat screen. Beware, other people looking at the featured image may not claim to see  $3 \times 10^{65}$  bits -- they might claim to see a teapot.

Image Credit: Caltech

**Download image:**  
teapotsirids\_winfree\_g6o.jpg

**Published:**  
Sunday, April 23, 2017 - 01:00

: , ; [ 4] [9] , NRF (2022)

가 가 .  
 ,  
 ,  
 (General Relativity)

가  
 ,  
 ,  
 가 ;

< 2> 252 :  
 vs. (가 : 가 (  $9.8m/s^2$ ), : 1 ,  
 100% 가 )

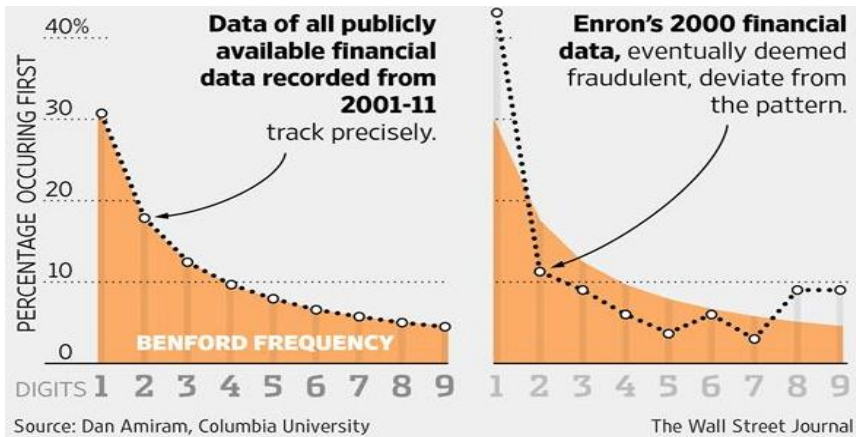
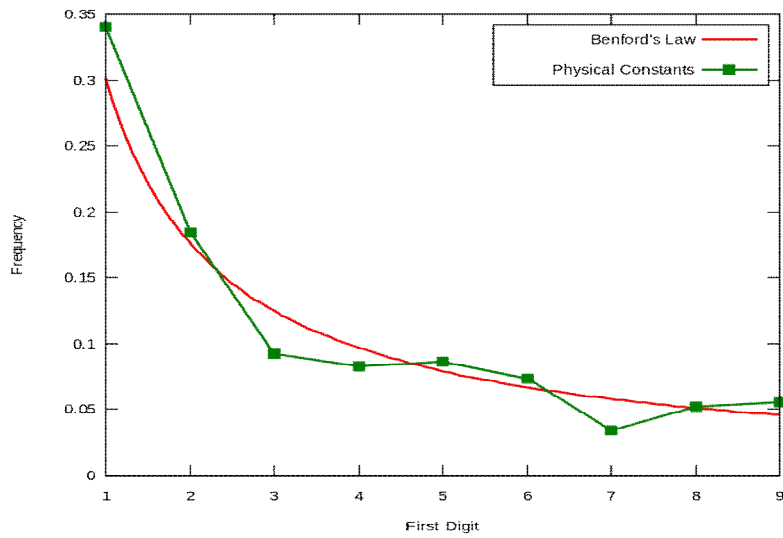
	15	2210
	252	2210
	520	260
	$2.99 \times 10^8$ m/s	$2.28 \times 10^{11}$ m/s

가 .  
 (Arithmetic Operator) 가  
 (Simon Newcomb, 12 March  
 1835 - 11 July 1909) (Frank Albert Benford Jr., 10 July 1883 -  
 4 December 1948) - (Newcomb-Benford  
 Law; First-digit Law; Law of Anomalous Numbers)

가 가 .  
 [ 5] 가 .

가 , 가 , 가 [10].

[ 5] , 가



[ 5] , [10][11]

가  
 ? (Product Operator)  
 (Multiplicative Fluctuations)  
 , “ × 가 ”  
 가  
 가  
 가  
 (Benford’s Law Compliance Theorem)  
 (Logarithm) (Fourier Transformation) 0  
 0 가 ,  
 ,  
 가 10  
 가 10 ,  
 , 가  
 가  
 (Number)  
 가 ,  
 (Product Operator)  
 ,  
 “ ”  
 “ ”

(String Theory) (Tensor Product)  
 가 ( )  
 ); ( 가 )  
 , “ ”  
 (Paul Adrien Maurice Dirac OM FRS, 8 August  
 1902 - 20 October 1984)  
 (Quantum Tunneling Effect)  
 (Electron)가  
 (Dirac Equation)(  
 가 )  
 가 (Topological  
 Insulator) (Superconductor)  
 (Majorana Fermion) ( )  
 , “ 가”  
 가 ,  
 (Real Number)가 (Complex Number)  
 (Quantum Field Theory) (Analytic Continuation)  
 (Renormalization) , “ ” “ ”  
 “ (?)” . 20~21 ,  
 (Number Field)  
 (Real Number Field)가 (Complex Number Field) . 가

, (Complex Vector Space) (Wave  
 Function) , (Matrix) (Tensor Product)  
 (Hamiltonian), (Eigenvector)  
 (Linear Combination) .  
 , (Product Operator)  
 . (⊗)  
 가 가 .

### III.

:

.

#### 1

(Hybrid) .

21 가

[13]

: GPU, AI,

가 /

(Newton's Method),

(Monte Carlo Method)

(Counting Frames) 가

2700

1970

17 6 가

17

(Pascal) 가

(Jacquard Loom)가 . 19

(Charles Babbage) (Differential Engine)

, 19 (Herman Hollery)

/ .

20 . 20

(Differential Analyzer)가 ,

(Alan Turing)

2

.

“

가?”( (Halting Problem))

[14].

(Gödel's Incompleteness Theorems)(

, )

, . ,

“ ” “ ” .

, “ ” ,

가 . [13].

“

가

, , ,

”

가 가 ,

(Universal Turing Machine)

, 가

(Stored-program Computer)

. ,

(John von Neumann) ,

, (

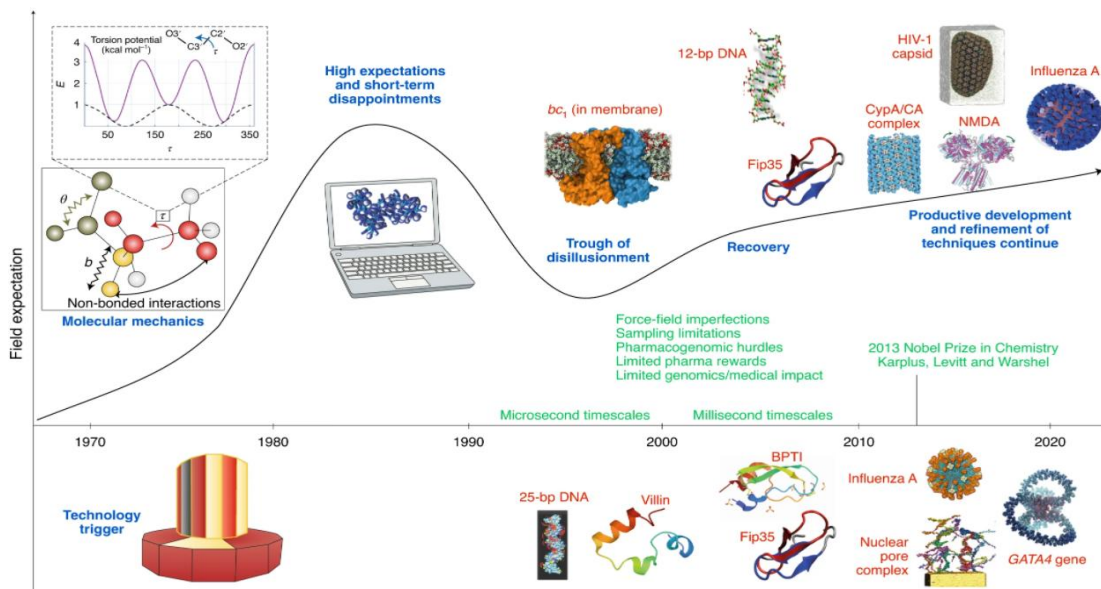
).

1

CPU

4

[13].



*	
25-bp DNA (5 ns, ~21,000 atoms)	villin protein (1 μs, 12,000 atoms)
bc1 membrane complex (1 ns, ~91,000 atoms)	12-bp DNA (1.2 μs, ~16,000 atoms)
Fip35 protein (10 μs, ~30,000 atoms)	nuclear pore complex (1 μs, 15.5 million atoms)
Fip35, bovine pancreatic trypsin inhibitory (BPTI) proteins	(100 μs for Flip35, 1 ms for BPTI, ~13,000 atoms)
influenza A virus (1 μs, >1 million atoms)	N-methyl-D-aspartate (NMDA) receptor in membrane (60 μs, ~507,000 atoms)
tubular cyclophilin A/capsid protein (CypA/CA) complexes (100 ns, 25.6 million atoms)	HIV-1 fully solvated empty capsid (1 μs, 64 million atoms)
GATA4 gene (1 ns, 1B atoms)	influenza A virus H1N1 (121 ns, ~160 million atoms)

[ 6 ]

[16]

, 1950 (“ ” 가 ) “ ”

[15].

(Turing test)

[13]. 1956 , “ ”

1 (AI Boom) 3

가 1970  
가 2000 가 (Expert System)

21 GPU /

, Nature [16] [ 6] , “  
GPU  
( : Technological innovations that have helped drive the field include distributed computations and the advent of the use of GPUs for biomolecular computations)”

, 2020 GPU  
(e.g. (AlphaFold))

20 (Neural Network)  
가 (Difficulty)

(Deep Neural Network)

21  
가 , 21  
“Computational Power”

가

가  
(Insight)  
(Data Science)  
(Paradigm)

가 2010  
2010 GPU (e.g.  
CPU 5~10 )가

가 .

2010 GPU

가 . 가 (Linear Operation) GPU ;

5~10 ;

(※ 가

(Overcompleteness)

– “ ”).

/ , 가

, CPU GPU

(Deep Learning; DL) , 가 20

[13].

, GPU , 1 GPU

DL HPC .

, 21

GPU 가 , 21

– (Order of Magnitude ) 가

–

.

,

.

가

,

가 .

2

가 가

(Quantum Computing)

(Quantum Physics)

[38]

Photon) (光子, Photon Cannon) 가 “ ” 가

가 가 가 (Incident Light) (Photon)  $E = hf$  ;

. 1 ,

가  $m$  ( )가 가  $a$   $F = ma$

$F$   $s$   $W = Fs$   $h$

$E_{\text{사파의 에너지}} = E_{\text{사파의 위치에너지}} = mgh$  , 가  $E_{\text{사파의 위치에너지}}$ 가

$E_{\text{사파의 운동에너지}} = \frac{1}{2}mv^2$  ,  $mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$  가 (  $h$ 가 )

,  $E_{\text{사파의 에너지}} = \frac{1}{2}mv^2 + mgh$  .

$p = mv$   $V$  ,  $H(x,p) := \frac{|p|^2}{2m} + V(x)$  [ 3].

$t$   $m$  ( $m > 0$ )  $(x(t), p(t))$  .

2 ( linear)

)  $\psi(x,t) = e^{i(kx - \omega t)}$  ( ) -

$$E = hf = \hbar\omega,$$

$$p = \hbar k, \quad \psi(x,t) = e^{\frac{i}{\hbar}(px - Et)} \quad 2$$

;

$$\psi(x,t) = e^{i(kx - \omega t)}$$

(代入) . 1

$\psi(x,t)$  “ 가 ” ,

( )

[ 3].

(Non-local) 가 .

(Operator)가 [

$$3]. \quad \hat{E} \hat{E}\psi(x,t) = E\psi(x,t) \quad E = \hbar\omega$$

, 가  $\hat{E}$  1

가 ( [38] ).  $E = \hbar\omega$  .

,  $E = \hbar\omega$   $\psi(x,t)$   $\hat{E}$  ( ,

$\exists ((E, \psi), \hat{E}), \hat{E}\psi(x,t) = E\psi(x,t) = \hbar\omega\psi(x,t)$  ).

$\psi (|\psi|^2 := 1$  [부록3] 참고) 4- 0

(m=0) . ,

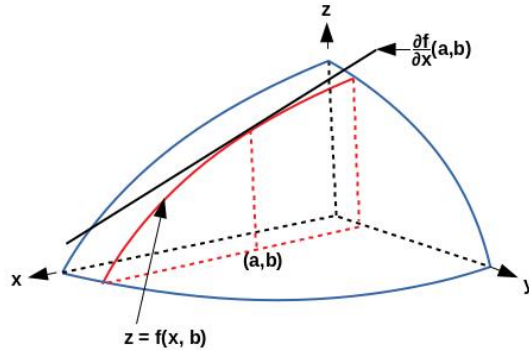
$$\psi(x,t) = e^{-ik_x x^\alpha} = e^{i(kx - ct\sqrt{kk + (\frac{mc}{\hbar})^2})} = e^{i(kx - \omega t)} = e^{\frac{i}{\hbar}(px - Et)} \left( kk = \left(\frac{\omega}{c}\right)^2, E = \hbar\omega \right)$$

( x t  $\Leftrightarrow$   $x^\alpha$ ),  $\hat{E}$  .

$$\hat{E} e^{\frac{i}{\hbar}(px - Et)} = E e^{\frac{i}{\hbar}(px - Et)} = -\frac{\hbar}{i} \frac{\partial}{\partial t} e^{\frac{i}{\hbar}(px - Et)} \Rightarrow \hat{E} = i\hbar \frac{\partial}{\partial t}$$

$$, \psi(x,t) = e^{\frac{i}{\hbar}(px - Et)} \quad \hat{E} = i\hbar \frac{\partial}{\partial t} \quad 2$$

, [ 7]



[ 7 ]

$$H(x, p) := \frac{|p|^2}{2m} + V(x)$$

; 0 (Case)  $m \approx 0 (m > 0)$ , 가  
[ 3 ],

$$\psi(x, t) \approx e^{-ik_0 x^\alpha} = e^{i(kx - ct \sqrt{kk + (\frac{mc}{\hbar})^2})} \approx e^{i(kx - \omega t)} = e^{\frac{i}{\hbar}(px - Et)} \left( kk = \left(\frac{\omega}{c}\right)^2, E \approx \hbar\omega \right)$$

$$\Rightarrow \hat{E} = E = H = \hat{H} \Rightarrow \hat{H}\psi(x, t) = \hat{E}\psi(x, t) \Rightarrow \hat{H}\psi(x, t) = i\hbar \frac{\partial}{\partial t} \psi(x, t)$$

$$V(x) \hat{V}\psi(x, t) = V\psi(x, t) \quad V$$

,  $V(x)$  ( ) ,  $\hat{V} = V(x)$

. ,  $\hat{V}\psi(x, t) = V(x)\psi(x, t)$ .

,  $p$  가 .  $p = \hbar k$  ;

$$\hat{P}\psi(x, t) = p\psi(x, t) = \hbar k\psi(x, t) \quad \hat{P}가 . , \hat{P}$$

$$\hat{P} e^{\frac{i}{\hbar}(px - Et)} = p e^{\frac{i}{\hbar}(px - Et)} = \frac{\hbar}{i} \frac{\partial}{\partial x} e^{\frac{i}{\hbar}(px - Et)} \Rightarrow \hat{P} = \frac{\hbar}{i} \frac{\partial}{\partial x}$$

0 (Case)  $m \approx 0 (m > 0)$ ,

$$\frac{1}{2m} \hat{P}\hat{P} \psi(x, t) = \frac{1}{2m} |p|^2 \psi(x, t) \Rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t) = \frac{1}{2m} |p|^2 \psi(x, t)$$

$$\hat{H}\psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t) + V(x)\psi(x, t)$$

$$\hat{H}\psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t), \quad \hat{\Xi}: \mathbb{C} \rightarrow \mathbb{C}, s = \hat{\Xi}\psi(x,t)$$

(Zero) (Schrödinger's Equation: ) ;

$$\hat{\Xi}\psi(x,t) = 0 \quad \text{where} \quad \hat{\Xi}\psi(x,t) := \left( i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} - V(x) \right) \psi(x,t).$$

, “ 가  $[\hat{X}, \hat{P}] = i\hbar$  ”  $\Rightarrow$  “ ”

\_\_\_\_\_ ( 가  $[\hat{X}, \hat{P}] = i\hbar$  ( \_\_\_\_\_ ), \_\_\_\_\_  
 \_\_\_\_\_ ) [ 3].

,  $x$  가 . 3 ( 4 )

$$. H(x,p), \quad x \quad x := x(t)$$

,  $V(x) := V(x,t)$  :

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left( -\frac{\hbar^2}{2m} \nabla_x^2 + V(x,t) \right) \psi(x,t).$$

$x$  2 , ,

$$x := \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \Rightarrow \nabla_x^2 := \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}, \quad px := (p_1 \ p_2 \ p_3) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = p_1 x_1 + p_2 x_2 + p_3 x_3$$

. , ( 가 );  $x, t,$

$\psi(x,t)$  .  $\psi(x,t)$  (3 ,

$$\psi : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{C}, \quad \hat{P} : \mathbb{C} \rightarrow \mathbb{C} \quad , \quad \hat{P} \quad \hat{P} \quad \hat{E}$$

, [ 3].

$\mathbb{C}$  가

가 .

(Erwin Schrödinger, 12 August

1887 - 4 January 1961)

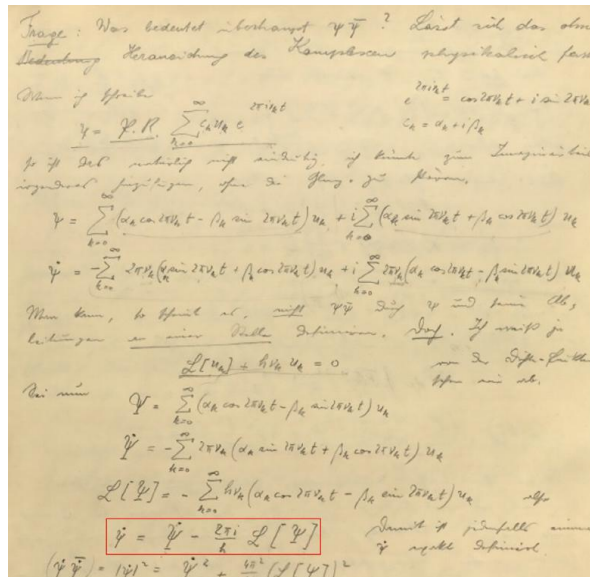
$\mathbb{C}$

“ $\psi\bar{\psi}$  가? 가 가?”  
 [ 8] - (David Hilbert,  
 January 1862 ~ February 1943)  
 [ 4]. C

[ 4].  
 “ 가 ,  
 ” 가  
 (Werner Karl Heisenberg, December 1901 ~ February  
 1976) (Uncertainty Principle) “

”  
 ;  
 “ 가 ”  
 ,  
 ( ) , 가  
 ; [ 3]

가 ,



[ 8]

[17]

( 化 ).  
 $AB \neq BA$   
 가 . 가 . ,  
 가 :  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ .  
 “ ” 가  
 “ ” A ,  
 B . , 가 가 ,  
 가 . , 가  
 가 ;  
 $A \neq B$  . 가 가 (Non-commutative Algebra)  
 . ,  
 ( 가 “  
 ” ).

$\hat{P}$   $\hat{X}$   
 $\hat{P}$   $\hat{X}$   
 ( ) 가 - ( )  
 가 . 가 .  
 , (※ 化 가 可).

$\begin{cases} f(x)=-x, g(x)=x^2 \Rightarrow f(g(x)):=f \circ g(x)=-x^2 \\ f(x)=-x, g(x)=x^2 \Rightarrow g(f(x)):=g \circ f(x)=+x^2 \end{cases} \Rightarrow f \circ g \neq g \circ f$   
 가 . , 가 .  
 ,  $x$   $f$   $-x$ 가 .  $g$   $x^2$ 가 . 가  
 ,  $x$   $g(x)$  ; ,  $g(-x)$  . ,  
 $g(x)=g(-x)$  . ,  $x$   $g$   $x^2$ 가 . 가 .  
 $f(x) \neq f(x^2) \Rightarrow -x \neq -x^2$  . . ,

$$f(g(x)) \neq g(f(x)) \Rightarrow f \circ g(x) \neq g \circ f(x) \Rightarrow -x^2 \neq x^2 .$$

가 化), “ ” 가 ( (=“ ”) 가 (Cheat Key)가 (Conservation) 가 “ ”

$$\begin{cases} f(x)=-x, g(x)=x^2 \Rightarrow f(x), g(x) \Rightarrow f(x), g(x) \\ f(x)=-x, g(x)=x^2 \Rightarrow g(x), f(x) \Rightarrow f(x), g(x) \end{cases} \Rightarrow (f(x), g(x))=(f(x), g(x))$$

( ) , ( ) 가 - 가 [ 3]. ;

가

“ ” 가 ; “ ”

가

가 ( )

가 ; ,

“ (e.g. , )”

가 ( ,

).

(Simulation)

( 가 )

$\Xi: \mathbb{C} \rightarrow \mathbb{C}, s = \Xi\psi(x,t) \Rightarrow \Xi\psi(x,t) = \xi\psi(x,t) (\xi \in \mathbb{C}),$  ( ,

$\hat{H}$  ) ;  $s = \Xi\psi(x,t)$

(Zero)

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \hat{E} \psi(x,t) = E_C \psi(x,t), \quad E_C \in \mathbb{C}$$

$$\left( -\frac{\hbar^2}{2m} \nabla_x^2 + V(x,t) \right) \psi(x,t) = \hat{H} \psi(x,t) \quad \text{양자컴퓨팅기반연산}$$

$\hat{H} \psi(x,t) = E_C \psi(x,t), E_C \in \mathbb{C}, \hat{H} \psi, \hat{\Xi} \psi$ : 양자중첩병렬 연산 ( $\hat{H} = \hat{E} - \hat{\Xi} \Rightarrow H = E_C - \xi$ )

\_\_\_\_\_ :  $E_C = E e^{i\theta} e^{i\alpha t}, \theta \rightarrow 0$  와  $\theta \pm \varepsilon$ 의 관심 반경 내 복소해석/대수기하적 구조 파악

$\hat{H} \psi$

가

(

),

(가

가

(Ground State)

)

.  $\hat{H} \psi$

가

;

,

,

,

$\hat{H} \psi$

.

,

,

“

(\_\_\_\_\_가

0

“

”

)”,

“

”

,

(

)

.

.

$x,$

$t,$

$\psi(x,t)$

$x$

$p$

$\langle x \rangle, \langle p \rangle$

.

,

$\psi(x,t)$

$(\langle x \rangle_{\psi(x,t)}, \langle p \rangle_{\psi(x,t)})$

;

가

가

- ,  $(\langle x \rangle_{\psi(x,t)}, \langle p \rangle_{\phi(x,t)=\hat{X}\psi(x,t) \neq \psi(x,t)})$ .

,

$\hat{X} \hat{P}$

(가

“

”

).

$\hat{X} \hat{P}$

-가

0

.

$\hat{X} \hat{P}$

가 0

.

,  $\hat{X} \hat{P}$

가 0

(

,

).

가 (“ ”) .

$\hat{I} \hat{I}\psi(x,t) := \psi(x,t)$  ,  $\hat{X} \neq \hat{I}$ ,  
 $\hat{P} \neq \hat{I}$  ,  $\hat{X} \neq \hat{P}$  ,  $\psi(x,t)$   
 $\hat{X}\psi(x,t) \neq \psi(x,t)$ ,  $\hat{P}\psi(x,t) \neq \psi(x,t)$ ,  $\hat{X}\psi(x,t) \neq \hat{P}\psi(x,t)$  .  
 ( )  $\psi(x,t)$   $\hat{X}\psi(x,t) = \phi(x,t)$  .  
 가 ,  $(\langle x \rangle_{\psi(x,t)}, \langle p \rangle_{\psi(x,t)})$   
 . ,  $(\langle x \rangle_{\psi(x,t)}, \langle p \rangle_{\phi(x,t) \neq \psi(x,t)})$   
 ,  $\hat{X}, \hat{P}$ 가 가  
 $(\langle x \rangle_{\psi(x,t)}, \langle p \rangle_{\phi(x,t) \neq \psi(x,t)}) = (\langle x \rangle_{\psi(x,t)}, \langle p \rangle_{\psi(x,t)})$ 가 ,  $\hat{X}, \hat{P}$   
 가 ,  $[\hat{X}, \hat{P}] := \hat{X}\hat{P} - \hat{P}\hat{X} = i\hbar$  가

(Non-local)

가

가 가 , 18

;

” , “MS ” 가 “MS + ”  
 ; “MS + ” “ ” “ 가 ”  
 - “MS ” “ ”

(Electron)

(Relativistic Effects)

(Dirac Equation),

(Quantum Electrodynamics; QED)

(※

가 (Speed of Light)

(Non-relativistic Limit;  $v \ll c$ )

[ 9]).

Electromagnetic interactions and the non-relativistic limit

Dirac Equation

Pauli Hamiltonian for the Schroedinger Quantum Mechanics

$$\left( i\hbar \frac{\partial}{\partial t} - q\phi \right) \Psi(\mathbf{r}, t) = (\alpha \cdot (-i\hbar \nabla - q\mathbf{A}) + \beta m) \Psi(\mathbf{r}, t) \quad \rightarrow \quad H_p \psi(\mathbf{r}) = \left( q\phi + \frac{[-i\hbar \nabla - q\mathbf{A}]^2}{2m} - \frac{q}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} \right) \psi(\mathbf{r})$$

[ 9]

(Dirac

Equation)

(Many-body)

3

( ) ,

(Quantum Computing)

가

가

$\Psi(x, t)$

,  $\Psi(x, t)$

$\psi(x, t)$

( )

$\psi(x, t)$

(※ 7/3 ;

“ (Coprime)

\_\_\_\_\_ 가” )

가 ,

가

. 가

(Hydrogen Atom)

1 \_\_\_\_\_

\_\_\_\_\_ 가

가 2 \_\_\_\_\_

(

), \_\_\_\_\_

가

$$\psi(x,t), \quad ( )$$

$$\Psi(x,t) = \sum_{\psi} c_{\psi} \psi(x,t)$$

(Hydrogen-like) ( “ ” )

(Spherical Coordinates)

(※ \_\_\_\_\_)

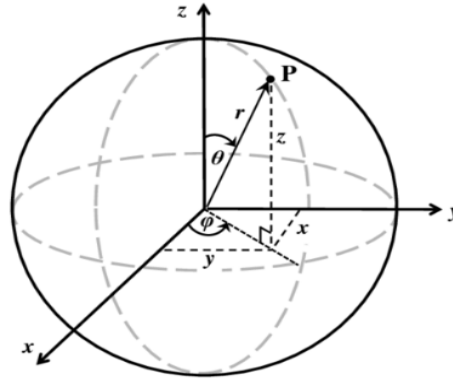
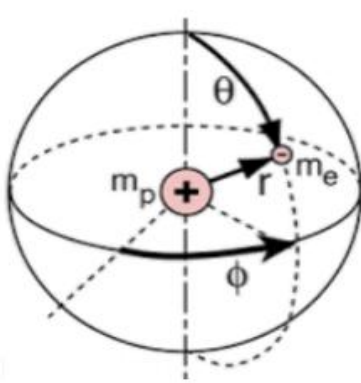
\_\_\_\_\_ ).

가

“ 가 ”

$$\psi(x,t)$$

$$\psi(x,t) := \psi(r, \theta, \phi, t)$$



$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}(z/r)$$

$$\phi = \tan^{-1}(y/x)$$

Figure Ref.: HyperPhysics (Prof. Carl R. Nave, Georgia State University) ※ [ 10]

$\phi = \varphi$

4-

4-

가  $\hat{H}$ 가  $4 \times 4$

$$i\hbar \frac{\partial}{\partial t} \psi(r, \theta, \phi, t) = \hat{H} \psi(r, \theta, \phi, t) = E \psi(r, \theta, \phi, t) \quad \text{with} \quad \psi(r, \theta, \phi, t) = R(r) Y(\theta, \phi) T(t) \quad \text{**} \quad T(t) = \exp\left(-\frac{iEt}{\hbar}\right)$$

$$\hat{H} = hc \begin{bmatrix} \frac{m_e c^2}{\hbar c} - \frac{Z\alpha_0}{r} & 0 & -i \cos \theta \left( \frac{k}{r} + \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right] \right) & -ie^{-i\phi} \sin \theta \left( \frac{k}{r} + \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right] \right) \\ 0 & \frac{m_e c^2}{\hbar c} - \frac{Z\alpha_0}{r} & -ie^{i\phi} \sin \theta \left( \frac{k}{r} + \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right] \right) & i \cos \theta \left( \frac{k}{r} + \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right] \right) \\ i \cos \theta \left( \frac{k}{r} - \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right] \right) & ie^{-i\phi} \sin \theta \left( \frac{k}{r} - \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right] \right) & -\frac{m_e c^2}{\hbar c} - \frac{Z\alpha_0}{r} & 0 \\ ie^{i\phi} \sin \theta \left( \frac{k}{r} - \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right] \right) & -i \cos \theta \left( \frac{k}{r} - \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right] \right) & 0 & -\frac{m_e c^2}{\hbar c} - \frac{Z\alpha_0}{r} \end{bmatrix}$$

(2-body Problem) , (Eigenvalue Problem)

(3-body Problem) 가 (Jules Henri Poincaré, 29 April 1854 - 17 July 1912; ) ,

(Many-body Problem) (Numerical Solution)

(Dominant) (Trial) (Iterative) ( )

$$\psi(r, \theta, \phi, t) = \frac{Ae^{-\frac{iEt}{\hbar}}}{r} \begin{pmatrix} G_{n,k}(r) \sqrt{\frac{k+\frac{1}{2}-m}{2k+1}} Y_k^{m-\frac{1}{2}}(\theta, \phi) \\ -G_{n,k}(r) \operatorname{sgn}(k) \sqrt{\frac{k+\frac{1}{2}+m}{2k+1}} Y_k^{m+\frac{1}{2}}(\theta, \phi) \\ iF_{n,k}(r) \sqrt{\frac{-k+\frac{1}{2}-m}{-2k+1}} Y_{-k}^{m-\frac{1}{2}}(\theta, \phi) \\ -iF_{n,k}(r) \operatorname{sgn}(k) \sqrt{\frac{-k+\frac{1}{2}+m}{-2k+1}} Y_{-k}^{m+\frac{1}{2}}(\theta, \phi) \end{pmatrix}, \quad E = \frac{m_e c^2}{\sqrt{1 + \left( \frac{Z\alpha_0}{n-|k| + \sqrt{k^2 - Z^2(\alpha_0)^2}} \right)^2}}$$

$$Y_a^b(\theta, \phi) = (-1)^{\max\{b,0\}} \sqrt{\frac{(2a+1)(1-|b|)!}{4\pi(1+|b|)!}} P_a^b(\cos \theta) e^{ib\phi} \quad P_a^b(x): \text{Associated Legendre Polynomial}$$

$$\rho = Cr, \quad C = \frac{\sqrt{m_e c^2 - E^2}}{\hbar c}, \quad s = \sqrt{k^2 - Z^2(\alpha_0)^2}, \quad \alpha_0 = \frac{e^2}{4\pi\epsilon_0 \hbar c}, \quad j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots \quad \left( s_{\text{Electron}} = \frac{1}{2} \right)$$

$$A = \frac{2^s}{\sqrt{sk(k-s)}} \sqrt{\frac{C}{n-|k|+s} \frac{(n-|k|-1)!}{\Gamma(n-|k|+2s+1)} \frac{1}{2} \left( \left( \frac{Ek}{sm_e c^2} \right)^2 + \frac{Ek}{sm_e c^2} \right)} \quad k = \pm \left( j + \frac{1}{2} \right), \quad l = j \pm \frac{1}{2}, \quad l < n \in \mathbb{N}$$

$$G_{n,k}(r) = \rho^s e^{-\rho} \left( Z\alpha_0 2\rho L_{n-|k|-1}^{2s+1}(2\rho) + (s-k) \frac{sm_e c^2 - kE}{\hbar c C} L_{n-|k|}^{2s-1}(2\rho) \right) \quad m = -j, -j+1, \dots, j-1, j$$

$$F_{n,k}(r) = \rho^s e^{-\rho} \left( (s-k) 2\rho L_{n-|k|-1}^{2s+1}(2\rho) + Z\alpha_0 \frac{sm_e c^2 - kE}{\hbar c C} L_{n-|k|}^{2s-1}(2\rho) \right) \quad L_a^b(x): \text{Associated Laguerre Polynomial}$$

$(\psi(r, \theta, \phi, t), E)$  .  
 $E$  ,  $\psi(r, \theta, \phi, t)$   $E$   
 $\psi(r, \theta, \phi, t)$  ,  
 $\psi(r, \theta, \phi, t)$   
 (Spinor Spherical Harmonics) (Radial Component)  
 “ (Product)” - “ ” ;  
 $1$  (Minimal Physical System)  
 “ (Quantum Entanglement)” , “ ” (Mathematical  
 Representation) ( , , ). 가  
 가 “ ”  
 “ ” “ ”  
 “ ” ( $\otimes$ )  
 가  $(r, \theta, \phi, t)$  “ ”  
 “ ” ,  
 (\*) 가  
 [ 3] ).

$E$  ,  $\psi(r, \theta, \phi, t)$  ,  
 “ ”  $\Psi(r, \theta, \phi, t) = \sum_{\psi} c_{\psi} \psi(r, \theta, \phi, t)$   
 - “ ” ,  $1$   
 $1$  ,  $\psi(r, \theta, \phi, t)$   
 가 “ ”  $\Psi(r, \theta, \phi, t)$  “ ” “ ”  
 가 “ ”  
 $\Psi(r, \theta, \phi, t)$  “ ” “ ” “ ”  
 $\Psi = \sum_{\psi} c_{\psi} \psi$   
 가 .

$$[\hat{X}, \hat{P}] = \hat{X}\hat{P} - \hat{P}\hat{X} = i\hbar \neq 0$$

; , 0

가

$$[\hat{L} \cdot \hat{S}, \hat{L}^2] = [\hat{L} \cdot \hat{S}, \hat{S}^2] = 0$$

가

;

$$\hat{L} \cdot \hat{S}\psi = a\psi$$

$$\hat{L} \cdot \hat{S}\hat{L}^2\psi = \hat{L}^2\hat{L} \cdot \hat{S}\psi = a\hat{L}^2\psi$$

가

a

$$\hat{L} \cdot \hat{S}\psi = a\psi$$

$$\hat{L} \cdot \hat{S}(\hat{L}^2\psi) = a(\hat{L}^2\psi)$$

가

$$\hat{L}^2\psi = b\psi$$

bψ 가

$$\hat{L} \cdot \hat{S}\psi = a\psi$$

a

$$\hat{L} \cdot \hat{S}(b\psi) = a(b\psi)$$

(※ Simultaneous Eigenstates).

가 ( , 가 )

$$[\hat{X}, \hat{P}] = \hat{X}\hat{P} - \hat{P}\hat{X} = i\hbar \neq 0$$

$$[\hat{L} \cdot \hat{S}, \hat{L}_z] = i\hbar \epsilon_{zki} \hat{L}_k \hat{S}_i$$

( $\epsilon_{ijk}$ : ; Permutation Tensor: Levi-Civita

Symbol)

$$[\hat{L} \cdot \hat{S}, \hat{L}_z] \neq 0$$

$$\psi(r, \theta, \phi, t)$$

( 가 )

가

$$(\hat{L} \cdot \hat{S})\psi(r, \theta, \phi, t) = \phi(r, \theta, \phi, t)$$

$$\psi(r, \theta, \phi, t)$$

$$\langle \hat{L} \cdot \hat{S} \rangle_{\psi(r, \theta, \phi, t)}, \langle \hat{L}_z \rangle_{\phi(r, \theta, \phi, t) \neq \psi(r, \theta, \phi, t)} \neq \left( \langle \hat{L} \cdot \hat{S} \rangle_{\psi(r, \theta, \phi, t)}, \langle \hat{L}_z \rangle_{\psi(r, \theta, \phi, t)} \right)$$

$$\left( \langle \hat{L} \cdot \hat{S} \rangle_{\psi(r, \theta, \phi, t)}, \langle \hat{L}_z \rangle_{\psi(r, \theta, \phi, t)} \right)$$

( “ ”

).

가

(Essence)

(Product)

가

(Tensor Product)

가

가 .  $[\hat{L} \cdot \hat{S}, \hat{L}_z] \neq 0$

$\psi(r, \theta, \phi, t)$  [ 5].

(Quantum Entanglement)

가 -

$(1, 0, 0) + 2(0, 1, 0)$  ,  $(1, 2, 0)$  3

Basis) ( ; Linearly Independent) (Orthonormal)

$\hat{H}$ 가 (

가 ; (가

).

$\hat{H}$   $\hat{H}_i$

( (Similarity Transformation)

(Block Diagonalization) ). (Solution)

( , )

가 , 가  $\frac{Y_X}{r} - \frac{aY+bX}{r}$

가 “ r” 가 ; r

가 / 가  $Y_X$  ).

가 , 3 (3, 0,

0)  $3(1, 0, 0)$  ,  $2(1, 0, 0) + a(0, 1, 0) + b(0, 0, 1)$

$a=b=0$  가 ,

“(1, 0, 0)” ,

.( “ ”) (Eigenstate) (Prime Number;

2, 3, 5, 7, 11, 13, 17, ...)

20

$$\Psi = \sum_{\psi} c_{\psi} \psi$$

$\hat{H}$

가

“ ”

$\Psi$

가

가  $\hat{L}_z \Psi$

“ ”

$$[\hat{L} \cdot \hat{S}, \hat{L}_z] = i\hbar \epsilon_{zki} \hat{L}_k \hat{S}_i$$

“ ”

,  $Y_{\chi}$

(Spinor)

$\chi$

가

,  $\hat{L}_z \Psi$

“ ”

“ ”

가

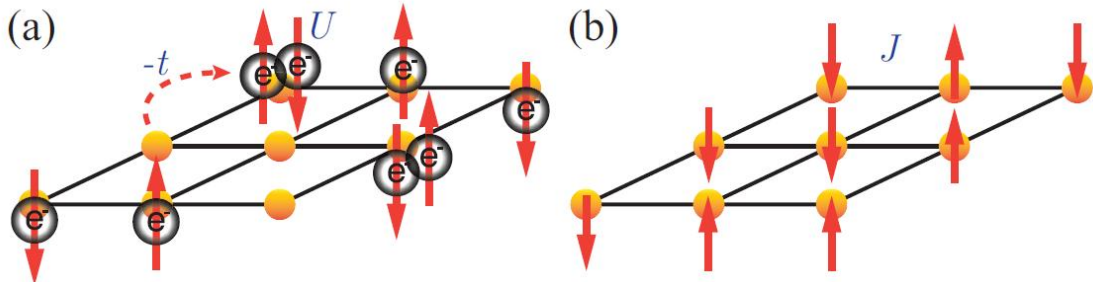


Figure: Schematic diagrams for (a) the Hubbard model on a square lattice with hopping  $-t$  and onsite energy  $U$  ( $e^-$ : electron,  $\uparrow$ : spin-up,  $\downarrow$ : spin-down) and (b) the spin-1/2 Heisenberg model on a square lattice with coupling  $J$ .

[ 11] (Lattice)

[ 11] (Electron) (Spin)

(Quantum Field Theory)

[ 12].

**t-J Model**

$$H_{t-J} = -t \sum_{\langle ij \rangle \sigma} (\hat{a}_{i\sigma}^\dagger \hat{a}_{j\sigma} + \hat{a}_{j\sigma}^\dagger \hat{a}_{i\sigma}) + J \sum_{\langle ij \rangle} \left( S_i \cdot S_j - \frac{n_i n_j}{4} \right)$$

(isotropic,  $J_x = J_y = J_z = J$ )

Real Solid

Minimal Model

Hubbard Model

Perturbative Expansion

t-J Model

$\sum_{\langle ij \rangle}$ : sum over nearest-neighbor sites  $i$  and  $j$   
 $\sum_{\sigma}$ : sum over spin polarizations  
 $S_k$ : spin on site  $k$  where  $S = S^x \hat{x} + S^y \hat{y} + S^z \hat{z}$   
 $\hat{a}_{i\sigma}^\dagger$ : fermionic creation operator  
 $\hat{a}_{j\sigma}$ : fermionic annihilation operator  
 $t$ : hopping integral  
 $J$ : coupling constant ( $= 4t^2/U$ ,  $U$ : Coulomb repulsion)  
 $n_k$ : number of particles at site  $k$  ( $= \sum_{\sigma} \hat{a}_{k\sigma}^\dagger \hat{a}_{k\sigma}$ )

[ 12] (Many-body System)

System) ; (Many-body System)

“ ” (Approximation) ,

2

$S^x, S^y, S^z$  -X, Y, Z

1/2 -1/2

### Heisenberg Spin-1/2 Model (Consisting of $N$ Spin-sites)

$$H_{\text{Heisenberg}} = J \sum_{\langle ij \rangle} S_i \cdot S_j$$

$$= J \sum_{\langle ij \rangle} \sum_{\alpha \in \{x,y,z\}} \bigotimes_{k=1}^N \{ (1 - \delta_{ik} - \delta_{jk}) I + (\delta_{ik} + \delta_{jk}) S^\alpha \}$$

$$\left( I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S^x = \frac{1}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad S^y = \frac{1}{2} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad S^z = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right)$$

$\delta_{ij}$ : Kronecker delta ( $\delta_{ij} = 1$  for  $i = j$ , otherwise  $\delta_{ij} = 0$ ); and  $\hbar = 1$ .

### Basis of Hilbert Space $\mathcal{H}$

1-Spin Site:  $\{|\uparrow\rangle, |\downarrow\rangle\}$  where  $|\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  and  $|\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

2-Spin Chain:  $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$  (cf.  $|\uparrow\uparrow\rangle = |\uparrow\rangle \otimes |\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ )

4-Spin Lattice ( $2 \times 2$ ):  $\{|\uparrow\uparrow\rangle, \dots, |\downarrow\downarrow\rangle\}$  (dimension: 16)

[ 13] t-J

[ 13] 2x2

4- (4-Spin Lattice) (Basis) 1

(Tensor Product;  $\otimes$ ) ;

“ ”

[ 13]  $H_{\text{Heisenberg}}$

$$H_{\text{Heisenberg}}($$

$$\hat{H} = H \quad (\text{Hat})$$

$H$ 가 가

### Basis of Hilbert Space $\mathcal{H}$

4-Spin Lattice ( $2 \times 2$ ):  $\{|\uparrow\uparrow\uparrow\uparrow\rangle, \dots, |\downarrow\downarrow\downarrow\downarrow\rangle\}$  (dimension: 16)

### Reduced Hilbert Space $\mathcal{H}_M$

$$\mathcal{H} = \bigoplus_{M=0}^4 \mathcal{H}_M$$

( $M$ : number of spin flips)

### Ground State

$|\phi\rangle \in \mathcal{H}_2$  exists such that

$$\langle\phi| H_2 |\phi\rangle = \langle\psi| H |\psi\rangle = E_{\text{ground state}}$$

( $|\psi\rangle$ : ground state,  $\langle v| = |v\rangle^\dagger$ ).

### Dimension of $H_2$ (Irreducible Matrix)

$$\dim(H_2) = 6 (< \dim(H) = 16)$$

( $\because$  matrix elements:  $\langle\psi_a| H_{\text{Heisenberg}} |\psi_b\rangle$  where

$$|\psi_a\rangle, |\psi_b\rangle \in \{|\uparrow\downarrow\uparrow\downarrow\rangle, |\uparrow\downarrow\downarrow\uparrow\rangle, |\uparrow\uparrow\downarrow\downarrow\rangle, |\uparrow\downarrow\uparrow\uparrow\rangle, |\uparrow\downarrow\downarrow\downarrow\rangle, |\downarrow\uparrow\uparrow\downarrow\rangle\}$$

[ 14]

(Irreducible Matrix)

$H_2$

[ 14] 16 , 6 (Reduced Hilbert Space) (Ground State Energy)

(Irreducible Matrix)  $H_2$

“ ” “ ”

(Irreducible Representation)

“ ” “ ”

( 가 가

).

[ 15]

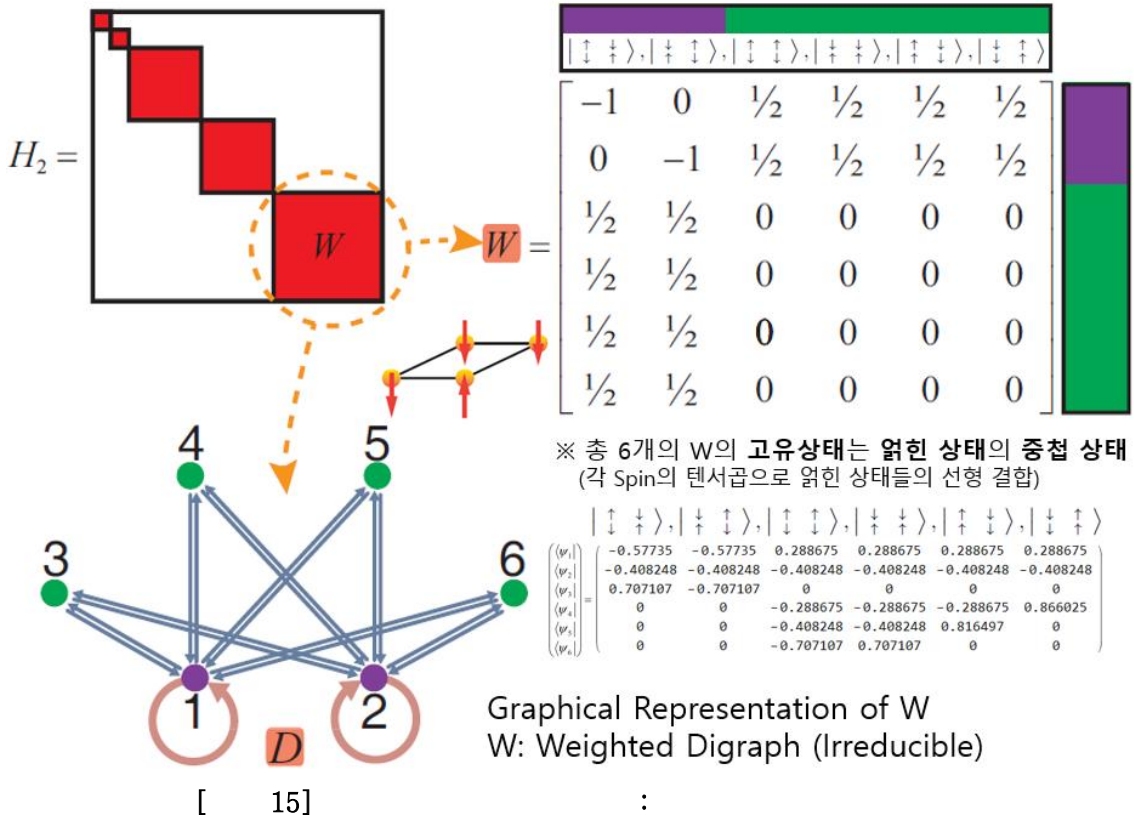
$H_2$

(Many-body)

(Interaction)

가 가

가



[ 15] 가 ( 가 ) (Standing Wave) ; 가 0 가 ) 가 (Weighted Digraph) [ 15] 가 (Algebraic Graph Theory) 가 가 (Irreducible) (Associated Matrix)

< 3 >

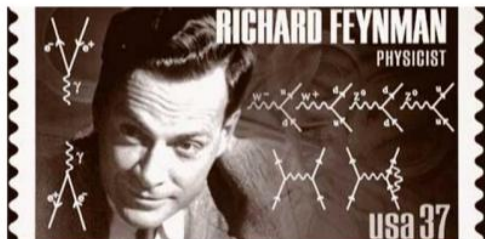
	$ \uparrow\uparrow\uparrow\rangle,  \uparrow\uparrow\downarrow\rangle,  \uparrow\downarrow\uparrow\rangle,  \uparrow\downarrow\downarrow\rangle,  \downarrow\uparrow\uparrow\rangle,  \downarrow\uparrow\downarrow\rangle$		
	$\begin{pmatrix} -1 & 0 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & -1 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \end{pmatrix}$	$ \psi_1\rangle$ $ \psi_2\rangle$ $ \psi_3\rangle$ $ \psi_4\rangle$ $ \psi_5\rangle$ $ \psi_6\rangle$	$\begin{pmatrix} -2. \\ -1. \\ 1. \\ 0 \\ 0 \\ 0 \end{pmatrix}$
	$ \psi_1\rangle,  \psi_2\rangle,  \psi_3\rangle,  \psi_4\rangle,  \psi_5\rangle,  \psi_6\rangle$		
( )	$\begin{pmatrix} -2. & 0 & 0 & 0 & 0 & 0 \\ 0 & -1. & 0 & 0 & 0 & 0 \\ 0 & 0 & 1. & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$ \psi_1\rangle$ $ \psi_2\rangle$ $ \psi_3\rangle$ $ \psi_4\rangle$ $ \psi_5\rangle$ $ \psi_6\rangle$	$\begin{pmatrix} -2. \\ -1. \\ 1. \\ 0 \\ 0 \\ 0 \end{pmatrix}$

,  $|\psi_i\rangle$  (i = 1, 2, 3, 4, 5, 6)  
 ( 1 ) < 3 > ( 1 )  
 \_\_\_\_\_ 가 \_\_\_\_\_

[ 15 ] .

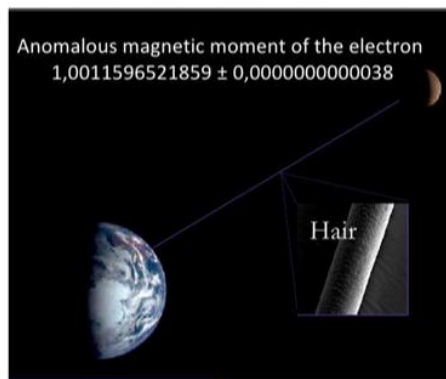
(Symmetry Group)

, 가 .



**QED**  
**THE STRANGE THEORY OF LIGHT AND MATTER**  
 Richard Feynman

Dirac's number (experiment) 1.00115965221  
 Dirac's number (theory) 1.00115965246



**Great Physicists** William H. Cropper

The theory of quantum electrodynamics is “the jewel of physics—our proudest possession,” Feynman writes in his fine book for the lay reader, *QED*. Few would disagree. The theory has been applied to a world of physical effects with complete success. Some of these phenomena permit measurements of astonishing accuracy, and calculations dictated by Feynman’s approach to QED are in agreement. For example, the strength of the magnetic field carried by an electron, the electron’s “magnetic moment,” has been measured as 1.00115965221 (in certain units), with an uncertainty of about 4 in the last digit. (This number is predicted to be exactly 1 in Dirac’s theory.) The theory calculates 1.00115965246, with an uncertainty of about 20 in the last digits, for the electron’s magnetic moment. If you could measure the distance from New York to Los Angeles with this accuracy, you would have it accurate to within the thickness of a human hair.

[ 16]

$(1, 0, 0)$   $(0, 2, 0)$  ,  $|\psi_i\rangle$  가 ( , ; e.g.  
 $(1, 0, 0)$   $(0, 2, 0)$  -  $( )$   $(0, 0, 0)$  ) ,  
 $|\psi_i\rangle$  가 가  
 (e.g.  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$  3  
 $\cdot \ast (1, 2, 3) = (1, 0, 0) + 2(0, 1, 0) + 3(0, 0, 1)$  );  
 (Basis) , (Orthonormalization)  
 (e.g.  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$  3 ).  
 (Index)가  $i$   $j$   
 $\hat{S}_i \cdot \hat{S}_j$  가 ,  $H_{\text{Heisenberg}}$  가

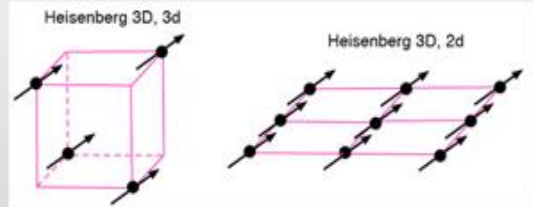
$2^n$  (Curse of Dimensionality)  
 $O(\exp(n)) \Rightarrow O(n)$   
 $\hat{S}_i \cdot \hat{S}_j$   
 (Spin-orbit Interaction)  $\hat{L} \cdot \hat{S}$   
 $\otimes$  (Spin-spin Interaction)  $\hat{S}_i \cdot \hat{S}_j$   
 $\hat{S}_i \cdot \hat{S}_j$ ,  $\hat{L}_i \cdot \hat{S}_j$  ( $\hat{H}$ )  $H_{\text{Heisenberg}}$   
 [ 16] : 0.00000001%  
 “LA 가 가 ”  
 LA,  
 [ 17],  $H_{\text{Heisenberg}}$  (Classical Statistical Mechanics)  
 $H_{\text{sing}}$ ,  $H_{\text{Heisenberg}}$   
 [ 12]  
 $\hat{S}_i \cdot \hat{S}_j$   
 20  
 “ ” (Mathematics) (Fact)

**Heisenberg model: D=3**

Spins are 3D vectors

The lattice dimensionality  $d=1,2,3...$

$$\hat{H}_{Heisenberg} = - \sum_{\langle ij \rangle} JS_i \cdot S_j$$

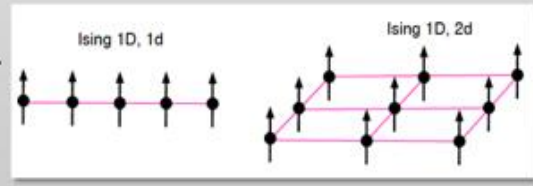


**Ising model: D=1**

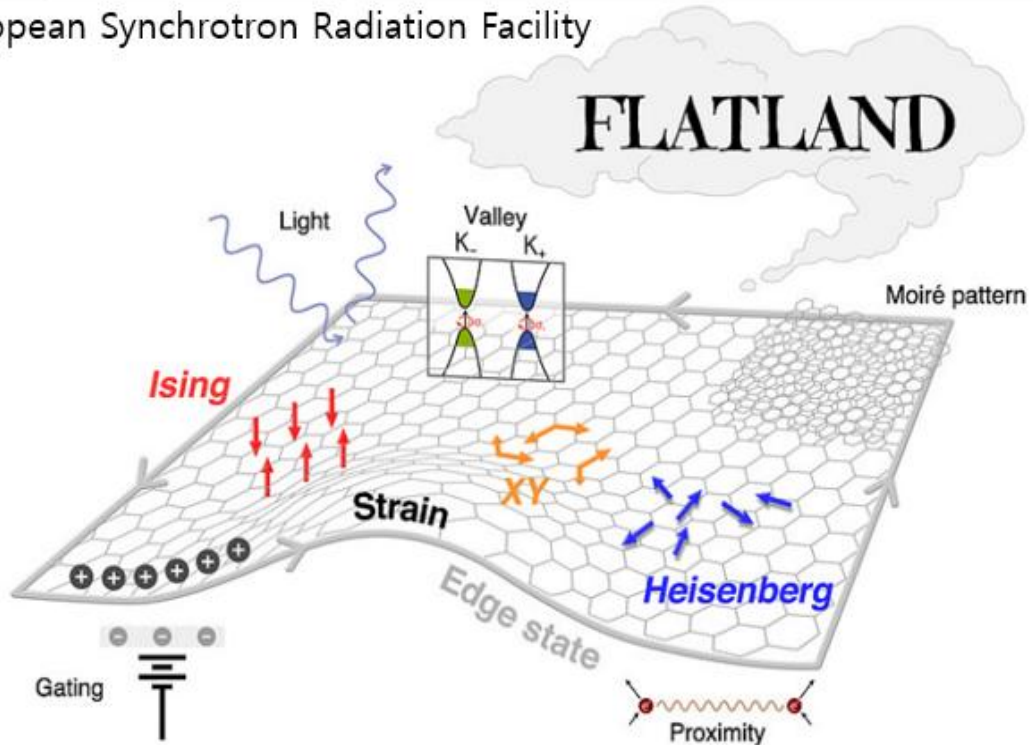
Spins are 1D vectors ( $S=S_z$ )

The lattice dimensionality can be  $d=1,2,3...$

$$\hat{H}_{Ising} = - \sum_{\langle ij \rangle} JS_i^z S_j^z$$



European Synchrotron Radiation Facility



▲ Figure 3: 2D van der Waals materials' roadmap. Beyond testing the theoretical models (Ising, XY, and Heisenberg), more application-driven tests for tuning the magnetism are also expected in the future. They could involve injecting electrons (gating), deforming the material (strain), placing different materials close together (proximity), controlling the angle at which magnetic layers are piled up (moiré pattern), and studying light and energy minima (spin-valley coupling).

Institute for Basic Science (IBS)

[ 17]

(Ising)

(Heisenberg)

가 가 3가  
 2가 가 : , , (Quantum  
 Teleportation) , ,  
 , 가  
 가  
 ,  
 .

4

가 , 가  
 , (Quantum  
 Cryptography) (Mathematica QuantumFramework , PC가 ).  
 (Protocol) ,  
 (Quantum Key Distribution) ; BB84  
 (BB84 protocol) (One-time Pad)  
 “ ”

- BB84 , 가
- 1) (Key)  $-X, Z$  (Pauli- $X, Z$  Basis Sequence)  
 (Random) .

- Key: [↑, ↑, ↓, ↓, ↑] → [|0>, |0>, |1>, |1>, |0>] in Pauli-Z Basis

- : [X, Z, Z, X, X]

2) < 4> , Key .

- : {  $\frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}$ , |0>, |1>,  $-\frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}$ ,  $\frac{|0\rangle}{\sqrt{2}} - \frac{|1\rangle}{\sqrt{2}}$  }

( , )	-Z	-X
↑>	$ \psi_{z_+}\rangle =  0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$ \psi_{x_+}\rangle = \frac{1}{\sqrt{2}}( 0\rangle +  1\rangle)$
↓>	$ \psi_{z_-}\rangle =  1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$ \psi_{x_-}\rangle = \frac{1}{\sqrt{2}}( 0\rangle -  1\rangle)$

3) 2) 5- (Qubit) (State) . , Key

(Tensor Product)

5-

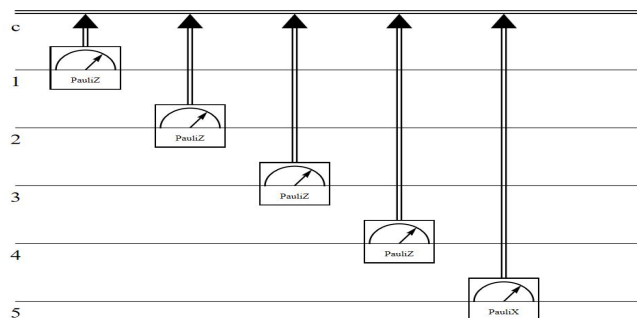
- 5- :  $-\frac{|00100\rangle}{2\sqrt{2}} - \frac{|00101\rangle}{2\sqrt{2}} + \frac{|00110\rangle}{2\sqrt{2}} + \frac{|00111\rangle}{2\sqrt{2}} - \frac{|10100\rangle}{2\sqrt{2}} - \frac{|10101\rangle}{2\sqrt{2}} + \frac{|10110\rangle}{2\sqrt{2}} + \frac{|10111\rangle}{2\sqrt{2}}$

4) 1)

- : [Z, Z, Z, Z, X]

5) 4) 3) 가 5-

[ 18]



[ 18] -X, Z

6) 가 5) 가 3) 5- (Tensor Product) ( , ) .

$$|\psi_{z_-} \psi_{z_+} \psi_{z_-} \psi_{z_+} \psi_{x_+}\rangle = |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes |0\rangle \otimes \left( \frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}} (|10100\rangle + |10101\rangle)$$

※ , .

$$\langle | \psi_{z_-} \psi_{z_+} \psi_{z_-} \psi_{z_+} \psi_{x_+} \rangle \rightarrow 0.25, | \psi_{z_-} \psi_{z_+} \psi_{z_-} \psi_{z_+} \psi_{x_+} \rangle \rightarrow 0.25, | \psi_{z_-} \psi_{z_+} \psi_{z_-} \psi_{z_+} \psi_{x_+} \rangle \rightarrow 0.25, | \psi_{z_-} \psi_{z_+} \psi_{z_-} \psi_{z_+} \psi_{x_+} \rangle \rightarrow 0.25 | \rangle$$

7) 6) Key . , Key -X, Z 가 -X, Z Key 가 .

- Key: [↓, ↑, ↓, ↑, ↑]

8) . , 6)

( , ) ( “ ” ) ( “ ” , ?).

- : [X, Z, Z, X, X] (Key: [↑, ↑, ↓, ↓, ↑])

- : [Z, Z, Z, Z, X] (Key: [↓, ↑, ↓, ↑, ↑])

- : [⊗, ○, ○, ⊗, ○]

- : [⊗, ↑, ↓, ⊗, ↑] → [↑, ↓, ↑] = [0, 1, 0]

- : [⊗, ↑, ↓, ⊗, ↑] → [↑, ↓, ↑] = [0, 1, 0]

\*\* , 가 , 1) ~ 8)

, BB84 , ; ; 가 . ( ) ( ) ( ) ( )

BB84 (One-time Pad) (Shor's Algorithm) (Order) (Quantum Polynomial Time)  $N$  [ 18] )

1. (고전 컴퓨터) 1과  $N - 1$  사이에 있는 자연수  $a$ 를 무작위로 고른다.
2. (양자 컴퓨터) 함수  $f(x) = a^x \pmod N$ 의 주기를 계산한다.
3. (고전) 주기  $r$ 가 짝수이면 (1)  $a^{r/2} - 1$ 와  $N$ 의 최대 공약수, 그리고 (2)  $a^{r/2} + 1$ 와  $N$ 의 최대 공약수를 구한다.

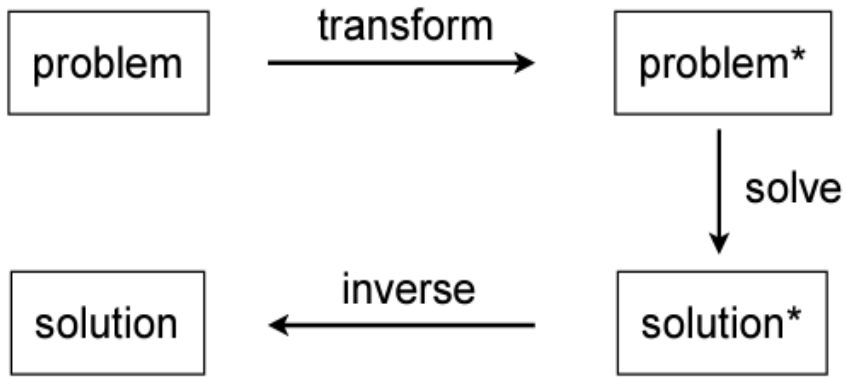
- 주기가 홀수이거나 최대 공약수가 1이나  $N$ 인 경우  $\rightarrow$  1로 돌아간다.
- 나머지 경우: 최대 공약수는  $p$  혹은  $q$ 와 일치하게 된다. [ 19] [18]

[ 19]

$r$

[ 20]

20]



**Classical**

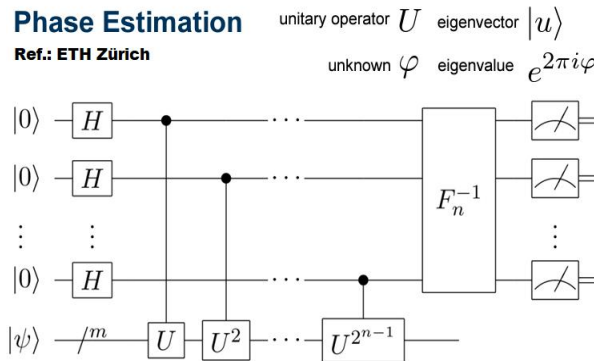
**Quantum**

$$y_k \equiv \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{2\pi i j k / N}$$

$$|j\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_j e^{2\pi i j k / N} |k\rangle$$

**Phase Estimation**

Ref.: ETH Zürich



1.  $|0\rangle |u\rangle$  initial state
2.  $\rightarrow \frac{1}{\sqrt{2^t}} \sum_{j=0}^{2^t-1} x_j |j\rangle |u\rangle$  create superposit
3.  $\rightarrow \frac{1}{\sqrt{2^t}} \sum_{j=0}^{2^t-1} x_j |j\rangle U^j |u\rangle$  apply black bo;  
 $= \frac{1}{\sqrt{2^t}} \sum_{j=0}^{2^t-1} x_j e^{2\pi i j \varphi_u} |j\rangle |u\rangle$
4.  $\rightarrow |\varphi\rangle |u\rangle$  apply inverse F

Figures Ref.: ETH Zurich

[ 20]

$N = 15, a = 7$ , [ 21] 7-  $N$   
 $|0000001\rangle$  7- -Z ( )  $\otimes$

$$|0000001\rangle = |0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |1\rangle$$

$$|0000001\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|0000001\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (\text{차원: } 2^7 = 128)$$

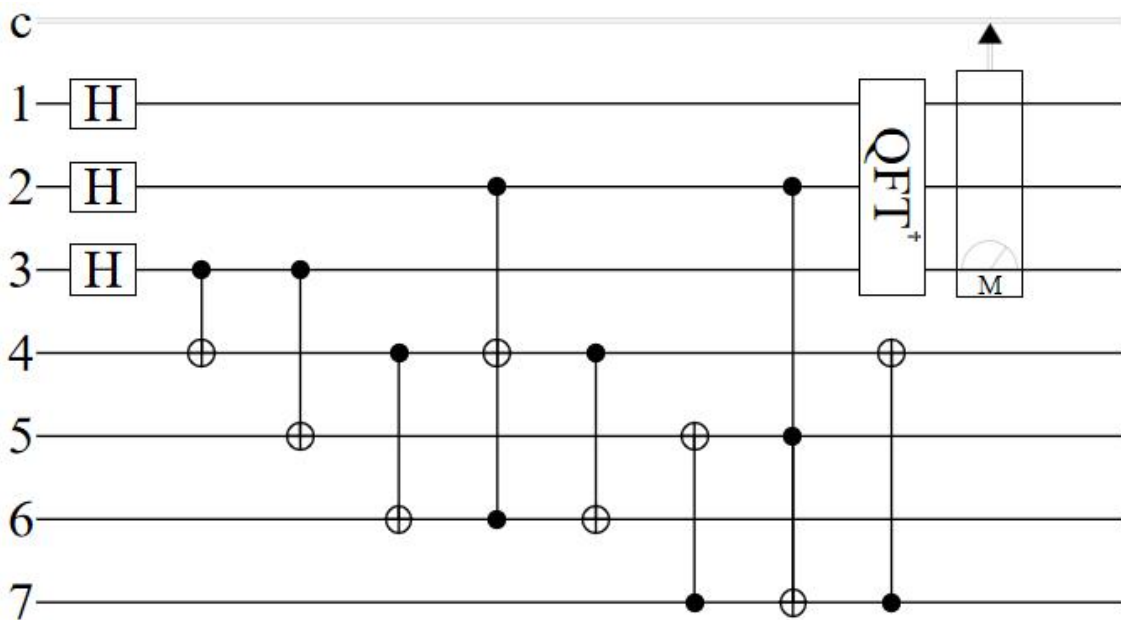
[ 21]  $\otimes$

, [ 21] 가 1, 2, 3

Hadamard H가 3, 4 ( 가 )

CNOT ●-○가 3, 5 가

가



[ 21]

[ 21]

가 .

(  $\otimes$  )  $\otimes$  ( ) ;

가

(Quantum Circuit Operation) ( ,

2  $\otimes$  2<sup>7</sup>

;  $\otimes$  ).

$$\text{Hadamard} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{Toffoli} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{QFT} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{-\frac{i\pi}{4}} & -i & e^{-\frac{3i\pi}{4}} & -1 & e^{-\frac{3i\pi}{4}} & i & e^{-\frac{i\pi}{4}} \\ 1 & -i & -1 & i & 1 & -i & -1 & i \\ 1 & e^{-\frac{3i\pi}{4}} & i & e^{-\frac{i\pi}{4}} & -1 & e^{-\frac{i\pi}{4}} & -i & e^{-\frac{3i\pi}{4}} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & e^{\frac{3i\pi}{4}} & -i & e^{\frac{i\pi}{4}} & -1 & e^{-\frac{i\pi}{4}} & i & e^{-\frac{3i\pi}{4}} \\ 1 & i & -1 & -i & 1 & i & -1 & -i \\ 1 & e^{\frac{i\pi}{4}} & i & e^{\frac{3i\pi}{4}} & -1 & e^{-\frac{3i\pi}{4}} & -i & e^{-\frac{i\pi}{4}} \end{pmatrix}$$

**Theorem:** Suppose  $s/r$  is a rational number such that

$$\left| \frac{s}{r} - \varphi \right| \leq \frac{1}{2r^2}$$

Then  $s/r$  is a convergent of the continued fraction for  $\varphi$ .

**Theorem:** if  $x$  non trivial solution of

$$x^2 = 1 \pmod{N}$$

Then at least either  $\gcd(x-1, N)$  or  $\gcd(x+1, N)$  is a non-trivial factor of  $N$ . Can be computed using  $O(L^3)$  operations.

**Theorem:**  $N = p_1^{\alpha_1} \dots p_m^{\alpha_m}$   
 $x$  chosen at random  $1 \leq x \leq N-1$  and co-prime with  $N$ .  $r$  is order of  $x$  mod  $N$ .  
 Then  $p(r \text{ is even and } x^{r/2} \not\equiv -1 \pmod{N}) \geq 1 - \frac{1}{2^m}$   
**Ref.: ETH Zürich**

[ 22]

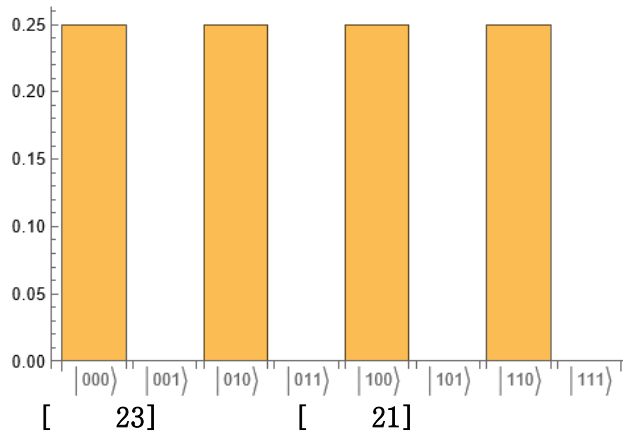
15, (Shor Circuit) / . [ 19] ,  $N=15, a=7$  , 가 CPU CPU 가 , 가

$|0000001\rangle$  ([ 21]) . 가 ;

(\*)

). [

21] (Inverse Fourier Transform) QFT† (Measurement) [ 23] .



[ 23] [ 21] , 75%

1 15 , 25%

$|000\rangle$  ,  $|000\rangle$

가 .  $\langle 5\rangle$  25% 75%

$N (N=15)$

< 5 > [ 22]

(CASE: a=7 → n=3)

	j(2^n/r) = s			GCD(15, a^{r/2}+1)	GCD(15, a^{r/2}-1)
	s	j	r		
000 >	0	0	r	?	?
010 >	2	1	4	5	3
100 >	4	1	2	1	3
110 >	6	3	4	5	3

※ GCD(A, B): A B

[ 21]

가 ,  
 ( ⊗ ) 가 ,  
 ;  
 |0000001 > , 가  
 .  
 .  
 7-  
 : 3- (Register) , 4-  
 (Ancilla) . 2^7=128 7-  
 ( 2^7=128 ( )  
 ⊗ )) ( , ) Ψ .  
 Ψ |000 > ⊗ |0000 > , ..., |111 > ⊗ |1111 > ,  
 7-  
 ; Ψ가 가 , 가 Ψ ,  
 ;  
 가 < 6 > . , 2021 Nature  
 [36] (N=21) .

< 6 >

$: H_1  \Psi\rangle$	
$\ast H_1  k_1\rangle \otimes  k_2 k_3 k_4 k_5 k_6 k_7\rangle = H_1  k_1 k_2 k_3 k_4 k_5 k_6 k_7\rangle$	
$ \Psi\rangle = \sum_{k_1=0}^1 \sum_{k_2=0}^1 \sum_{k_3=0}^1 \sum_{k_4=0}^1 \sum_{k_5=0}^1 \sum_{k_6=0}^1 \sum_{k_7=0}^1 c(k_1, k_2, k_3, k_4, k_5, k_6, k_7)  k_1 k_2 k_3 k_4 k_5 k_6 k_7\rangle$	
$\ast c(k_1, k_2, k_3, k_4, k_5, k_6, k_7)$	가
$2^7$	$H_1 c(k_1, k_2, k_3, k_4, k_5, k_6, k_7)  k_1 k_2 k_3 k_4 k_5 k_6 k_7\rangle$
$1$	$ \Psi\rangle \quad H_1  \Psi\rangle$
$\ast$	$: \sim O(\exp(N)), \quad : \sim O_{\text{양자}}(N)$
$\ast$	$N \quad \log$

< 6 >

N

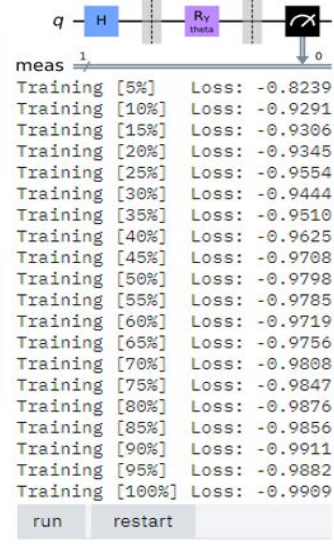
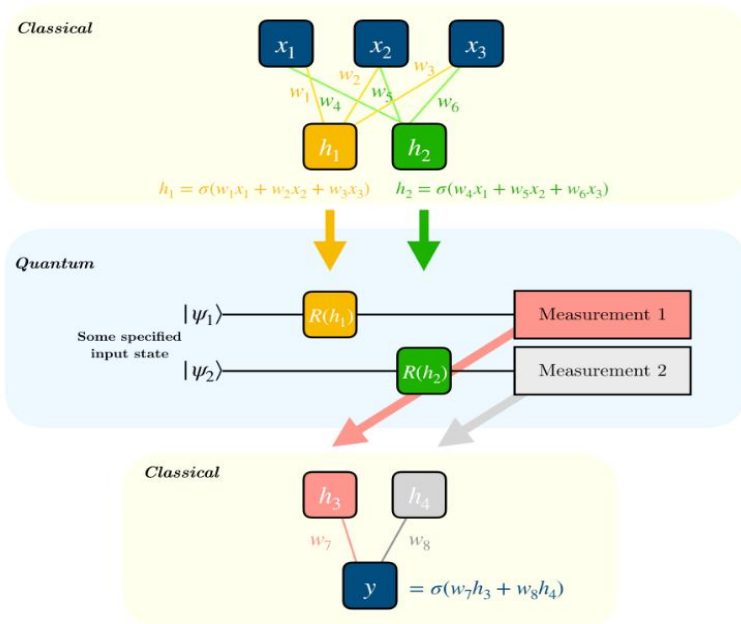
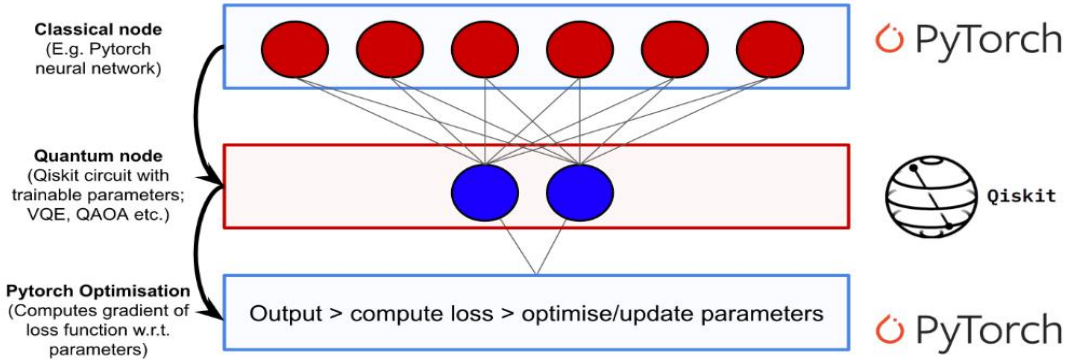
가

, IBM Qiskit (<https://qiskit.org>)

※

<https://qiskit.org/textbook/ch-machine-learning/machine-learning-qiskit-pytorch.html>

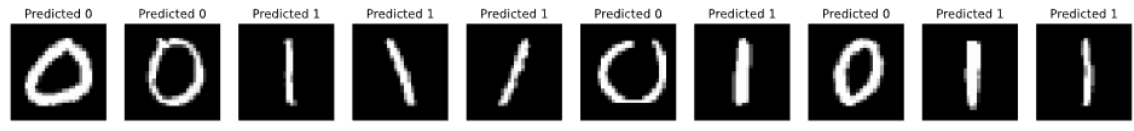
<https://qiskit.org/textbook/ch-applications/vqe-molecules.html>



Performance on test data:  
Loss: -0.9824  
Accuracy: 100.0%

$$\nabla_{\theta} \text{Quantum Circuit}(\theta) = \text{Quantum Circuit}(\theta + s) - \text{Quantum Circuit}(\theta - s)$$

가 (Test) (0 1 )



[ 24] IBM - (PyTorch and Qiskit)

IBM Qiskit

가

Mathematica

(Symbolic Operation)

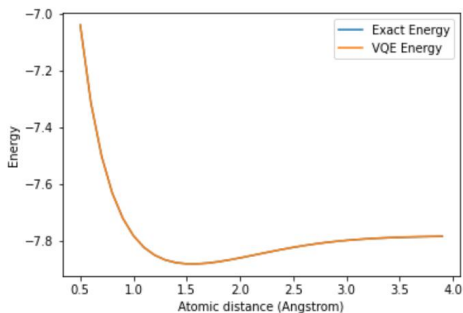
( / 要).

/ Run

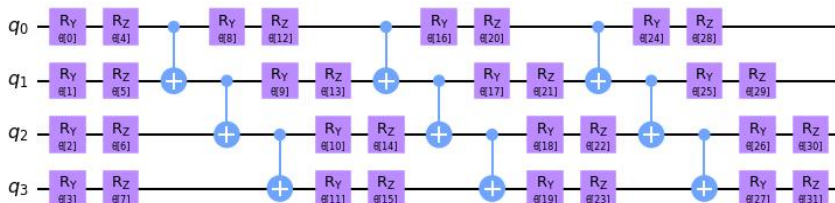
가

```

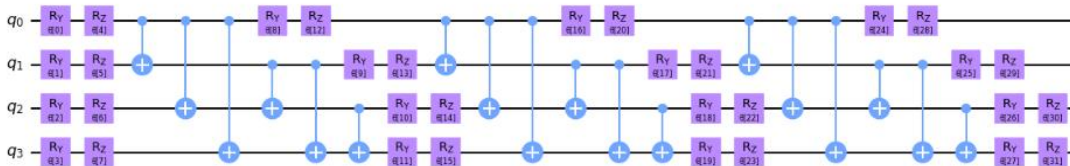
Interatomic Distance: 2.2 VQE Result: -7.844878726181193 Exact Energy: [-7.84487909]
Interatomic Distance: 2.3 VQE Result: -7.837257439443965 Exact Energy: [-7.83725797]
Interatomic Distance: 2.4 VQE Result: -7.8299358445800435 Exact Energy: [-7.829937]
Interatomic Distance: 2.5 VQE Result: -7.823078191944695 Exact Energy: [-7.82307664]
Interatomic Distance: 2.6 VQE Result: -7.816782592132487 Exact Energy: [-7.816779515]
Interatomic Distance: 2.7 VQE Result: -7.811153437640691 Exact Energy: [-7.81116828]
Interatomic Distance: 2.8 VQE Result: -7.80621829754091 Exact Energy: [-7.80622956]
Interatomic Distance: 2.9 VQE Result: -7.801962397987873 Exact Energy: [-7.8019736]
Interatomic Distance: 3.0 VQE Result: -7.798352411305384 Exact Energy: [-7.79836343]
Interatomic Distance: 3.1 VQE Result: -7.795326815002424 Exact Energy: [-7.79534045]
Interatomic Distance: 3.2 VQE Result: -7.792800696732087 Exact Energy: [-7.79283481]
Interatomic Distance: 3.3 VQE Result: -7.790603797885796 Exact Energy: [-7.79077401]
Interatomic Distance: 3.4 VQE Result: -7.788715355036123 Exact Energy: [-7.7889889]
Interatomic Distance: 3.5 VQE Result: -7.78721577056921 Exact Energy: [-7.78771697]
Interatomic Distance: 3.6 VQE Result: -7.786080395256832 Exact Energy: [-7.78660376]
Interatomic Distance: 3.7 VQE Result: -7.785203510040527 Exact Energy: [-7.78570291]
Interatomic Distance: 3.8 VQE Result: -7.784479536529381 Exact Energy: [-7.78497559]
Interatomic Distance: 3.9 VQE Result: -7.783853365368745 Exact Energy: [-7.78438961]
All energies have been calculated
    
```



====Linear Entanglement:====



====Full Entanglement:====



※

가

[ 25]

IBM Qiskit

(H<sub>2</sub>)

, IBM Qiskit  
 . , 가 ; ,  
 .  
 ,  
 ;

(Ideally)  
 ( -273.15°C) 가 ( ,

Classical Limit 가 [ 3])

; Classical Limit 가

.  
 ,  
 ,  
 ,  
 ,  
 가

가 ,  
 ; 가  
 ( , )

(Quantum Teleportation)

$\Psi$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle_{\text{영희}} \otimes |00\rangle_{\text{철수}} + |11\rangle_{\text{영희}} \otimes |11\rangle_{\text{철수}}) \text{ 혹은}$$

$|\Psi\rangle = |00\rangle_{\text{영희}} \otimes |00\rangle_{\text{철수}}$  (가장 단순한 양자 얽힘 기저 상태; 예시의 단순화)

[ 26]

$\Psi$

;  $\Psi$

, [ 26]

가  $\Psi$

가

가

,  $\Psi$

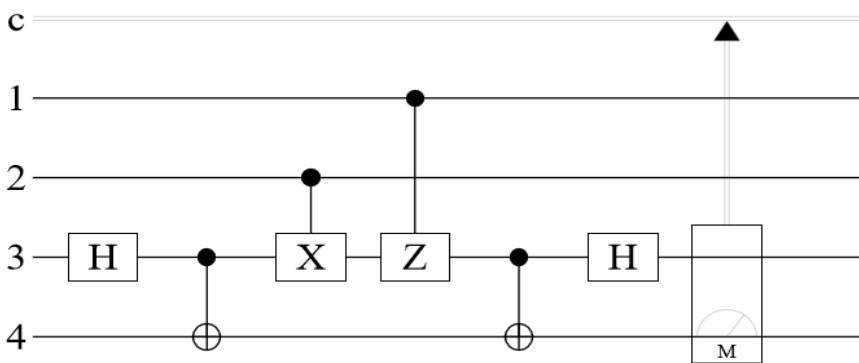
(

).

[ 26] 가 ;  
 [ 26] 3, 4 M(Measurement) 가 ( 가  
 ) . [ 21] , 가

$$|\Psi\rangle = |00\rangle_{\text{영희}} \otimes |00\rangle_{\text{철수}} = |0\rangle_{\text{영희}} \otimes |0\rangle_{\text{영희}} \otimes |0\rangle_{\text{철수}} \otimes |0\rangle_{\text{철수}} \quad < 7 >$$

00, 01, 10, 11



$$\text{Hadamard} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{CX} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{CZ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

[ 26]  $\Psi$

< 7 > ( )

가		$\Psi$	
		가	가 ( )
00	$I \otimes I \otimes I \otimes I$	$ 00\rangle_{\text{영희}} \otimes  00\rangle_{\text{철수}}$	$ 00\rangle_{\text{영희}} \otimes  00\rangle_{\text{철수}}$
01	$I \otimes \sigma_x \otimes I \otimes I$	$ 01\rangle_{\text{영희}} \otimes  00\rangle_{\text{철수}}$	$ 01\rangle_{\text{영희}} \otimes  01\rangle_{\text{철수}}$
10	$\sigma_x \otimes I \otimes I \otimes I$	$ 10\rangle_{\text{영희}} \otimes  00\rangle_{\text{철수}}$	$ 10\rangle_{\text{영희}} \otimes  10\rangle_{\text{철수}}$
11	$\sigma_x \otimes \sigma_x \otimes I \otimes I$	$ 11\rangle_{\text{영희}} \otimes  00\rangle_{\text{철수}}$	$ 11\rangle_{\text{영희}} \otimes  11\rangle_{\text{철수}}$

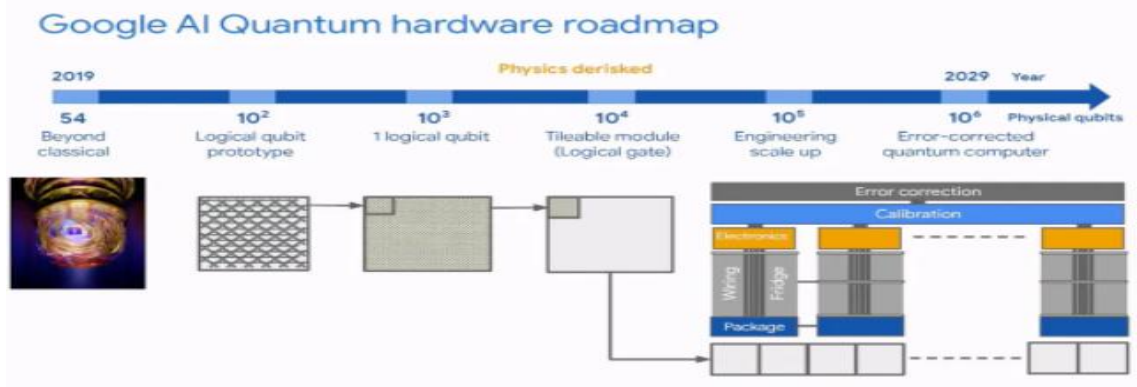
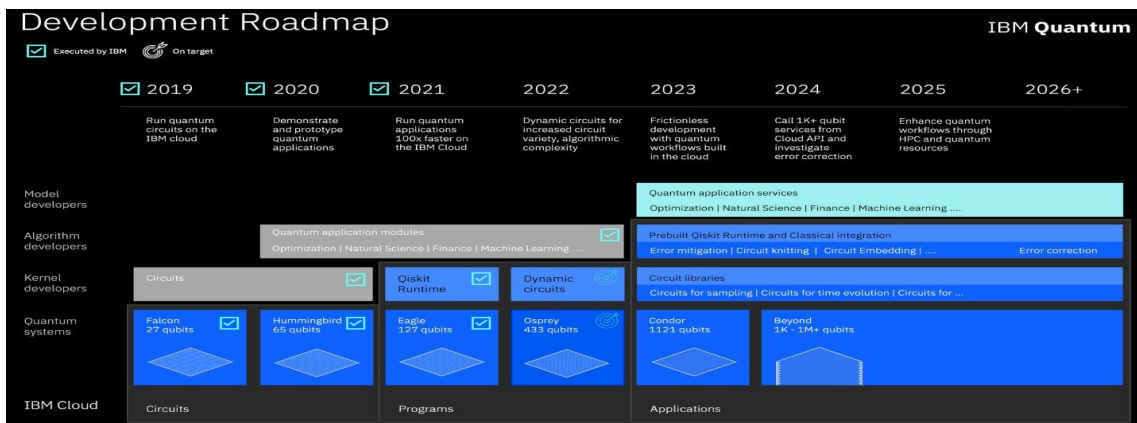
$\langle 7 \rangle$  가 ,  $\Psi$  [ 26]  
 가 . ,  
 ; 가  
 (= ) ,  
 “ ”( )  $\Psi$   
 가 . ,  
 EPR ( - - , ) ,  $\cdot CH \cdot CHSH$  ;  
 , ( ) 가 “ ( . ,  
 ; )” “ ”[ 3].  
 , , .  
 , 가 . .  
 가  
 . 가 ,  
 ; 가 20  
 , , .



, [ 27] 가 BQP  
 P ,  
 가 ,  
 가 가 .  
 가,  
 , NP-Complete (Polynomial Time)  
 ; ,  
 P=NP .  
 P≠NP가 P BQP (Task) BQP  
 (Blockchain)  
 가 가 .  
 (Game Theory) / 가가  
 , [ 27]  
 가  
 가 NP-Complete  
 O(exp(exp(exp(exp(n)))) 가 ,  
 O(exp(n)) ( : ↑ ↓ 2가 n  
 1 2^n ). , (Curse of  
 Dimensionality) , O(exp(n)) (  
 ); 가  
 (Ideal) 가 , 가  
 가 (Exponentially) 가 .

1

가 IBM  
 (Google) . 가  
 .  
 [ 28] IBM . IBM  
 2023 . IBM  
 가  
 . IBM , (Actual  
 Universe) 2.73°C , ( -273.15°C) 가  
 ( , Quantum Limit 가 )



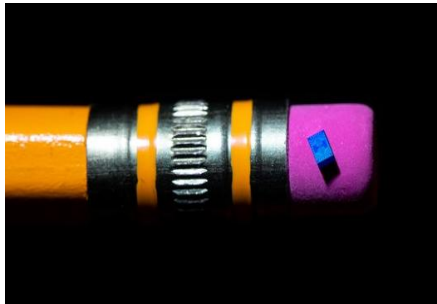
[ 28] IBM

2019 10 ' , , 가 가 가 Nature 가 1 3

(State-of-the-art; SOTA) (同) 3 가 (Topological Quantum Computer) 가

Nature 가 ( : Insufficient Scientific Rigor)

2021 10 IonQ (Trapped Ion)



[ 29]

95%

1

가

[19].

(Bridge) / (Hybrid) (NVIDIA) cuQuantum (Tensor Network) (State Vector) (Tool) . cuQuantum NVIDIA DGX A100 (Xanadu) (Quantum Chemistry) (Framework) (PannyLane) cuQuantum (Azure Quantum) 가 Q# Development Kit, QDK MRI 30% CPU, GPU, FGPA

## Amazon Braket

양자 컴퓨팅 연구 가속화

Amazon Braket 시작하기

**매월 1시간 무료 시뮬레이션 시간**

1년 동안 제공 AWS 프리 티어 혜택

일관된 개발 도구 집합을 사용하여 다양한 유형의 양자 컴퓨터와 순환 시뮬레이터로 손쉽게 작업합니다.

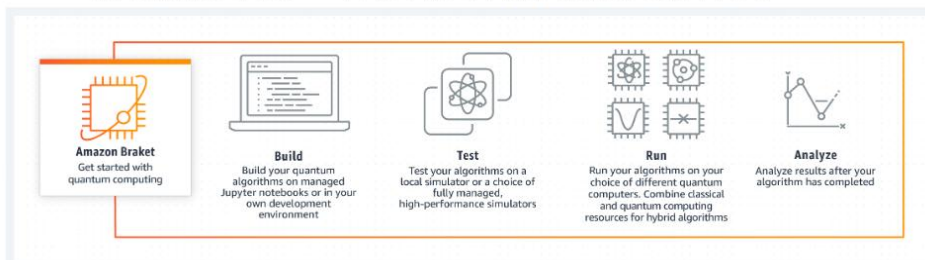
양자 및 클래식 워크로드 모두에 대한 간단한 요금 책정 및 관리 제어를 통해 신뢰할 수 있는 클라우드에서 양자 프로젝트를 구축하십시오.

관리할 기존 인프라 없이 양자 컴퓨터에 우선적으로 액세스하여 하이브리드 양자-클래식 알고리즘을 더 빠르게 실행합니다.

전문 안내와 기술 지원을 통해 빠르게 혁신하거나 Amazon Quantum Solutions Lab의 컨설턴트와 협업하세요.

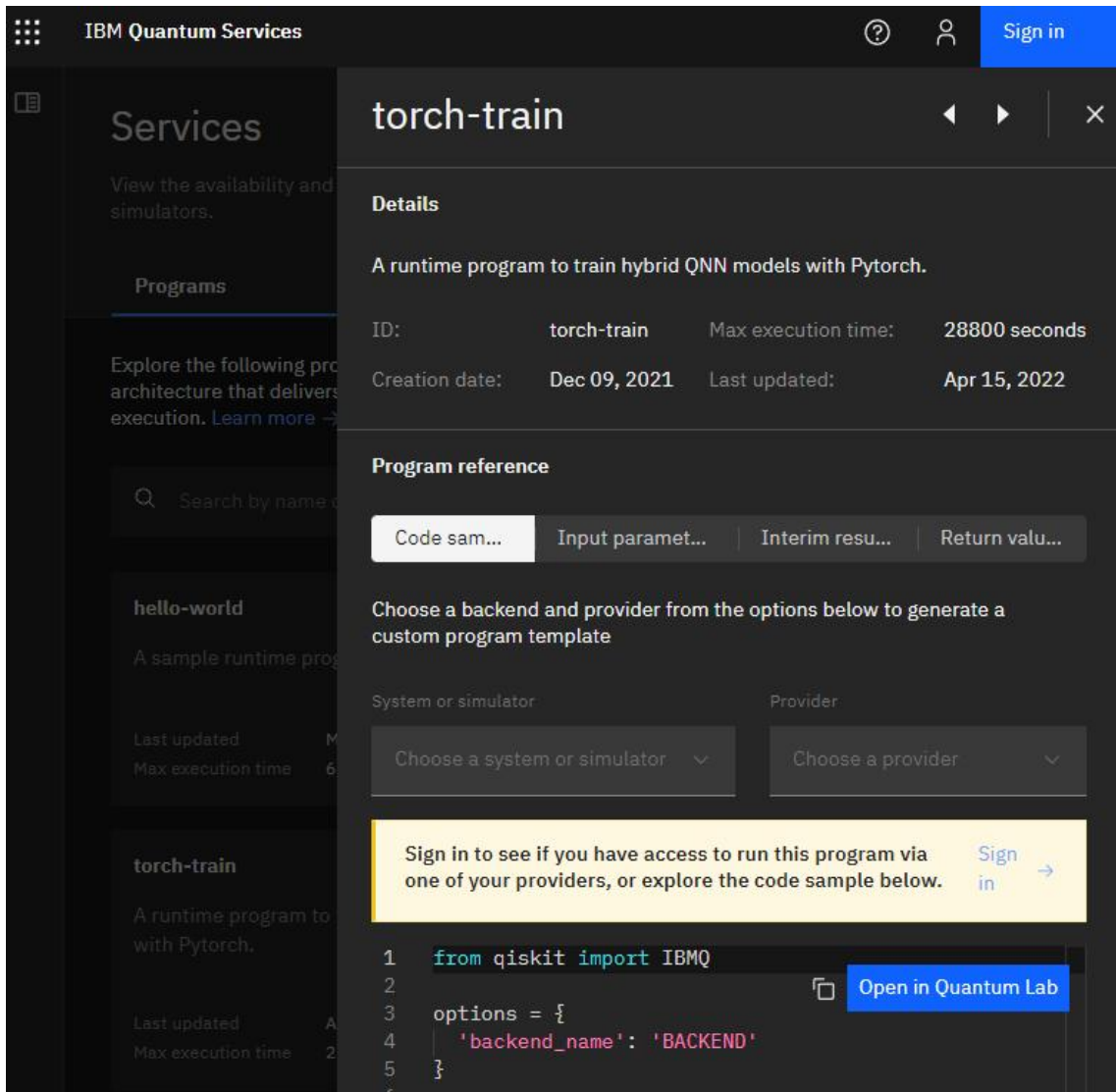
### 작동 방식

Amazon Braket은 양자 컴퓨팅을 위한 과학 연구 및 소프트웨어 개발 속도를 높이기 위해 설계된 완전관리형 양자 컴퓨팅 서비스입니다.



[ 30 ] (Amazon Braket)

(TensorFlow Quantum) , IBM Q Experience[ 31], Google Quantum AI, Xanadu Quantum Cloud, Forest, Amazon Braket[ 30], Azure Quantum, Quantum Inspire, Forge, D-Wave, AQT, Oxford Quantum Circuits, Alibaba Cloud, Quantinuum 가 (Baidu) 2020 (Paddle Quantum)



[ 31] IBM Quantum Services torch-train Hybrid QNN AI

(Hybrid)

가

가

가

가

가

가

가

“ (Mass)”

(“Velocity”)

(Upper Bound)가

“ (Momentum)”

/

※

:  $P_{\text{대한민국 양자기술 발전 모멘텀}} = M_{\text{양자기술자 그룹의 질량}} V_{\text{양자기술 실적 산출 속도}}$

$M_{\text{양자기술자 그룹의 질량}} \propto N_{\text{양자기술자의 수}} \bar{m}_{\text{양자기술자의 질량}}$

\* “ ” 가 “ , , , ”

2 - , , , EU, , , , , 11 가 ( “ ” ) 1 [20]. 가 ( ) 가 , 가 (‘19~) , (‘14~) , ( ) (‘19~) . 1 10~100 가 가 . 2021 2 18 가 . [ 32]



### 양자컴퓨터 관련 국가별 주요정책

- 미국** 2016년 양자정보화학 발전계획 발표  
2019년부터 5년간 최대 13억 달러(약 1조 6464억원) 규모 투자
- 중국** 양자컴퓨터와 양자통신 국가 중점 사업으로 선정  
2018년 이후 5년간 중국은 1000억 위안(약 17조원) 투입
- 유럽** 양자컴퓨터 공동 개발 위한 퀀텀 유럽 프로젝트 시행  
유럽연합(EU)은 양자컴퓨터를 유럽 주권 기술로 지정
- 일본** 양자컴퓨터 10대 중점 분야로 선정  
2000억엔(약 1조9421억원) 규모 프로젝트 가동
- 한국** 2022년까지 50큐비트급 양자컴퓨터 개발 목표  
2030년 양자 핵심인력 1000명 육성

자료: 각국 정부 발표 자료

22.05.02 전진우 그래픽 기사 618tue@newsis.com

가 [21]

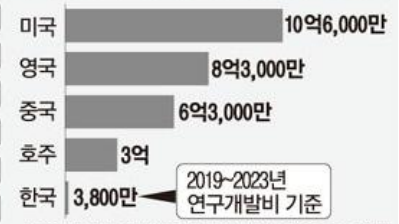
< 8 >

<p>[22]</p>	<p>* (NIST) ('17 ~)          * (2022.05.)          - 가 : 가 가          - : 가 가          - → 新          * '24 ~ '35          * ∞</p>
	<p>* ( , 2017)          - 5 18          * 14 5 (2021~2025) (2020)          - ,          * (USTC)          * 가</p>
	<p>* (2021.06.)          - , IBM          * 가 (2022.01.)          - D-WAVE , Jülich Supercomputing Centre          - 가 (e.g. )          * 5 2.7</p>
	<p>* (2021.11.)          - 2040 50%          *          *          - 가</p>
<p>EU</p>	<p>* (QuIC) (2021.04.)          - .          *          - 2028 1.3          - . .</p>
	<p>*          - (光) (2021.12.)          - 2030 , 2.1</p>

글로벌 주요 기업 양자컴퓨터 개발 개요

IBM	2016년부터 클라우드 방식 서비스 제공. 다임러, 웰스파고, 델타항공 등에 서비스
구글	하드웨어 개발·클라우드 서비스 준비 병행
MS	지난 2월부터 클라우드 서비스 '애저'에 양자컴퓨터 연구 플랫폼 준비
인텔	반도체 기반 양자 프로세서 개발에 집중
아마존	아마존웹서비스(AWS) 클라우드에 양자컴퓨팅 서비스 준비
알리바바	2018년부터 자체 클라우드에 연산을 직접 실행해볼 수 있는 서비스

국가별 양자컴퓨터 연구개발비 (단위=달러)



\*2009년~2018년까지 10년 연구개발비 기준. 자료=아스타뮤제

미국	중국	영국	핀란드	호주	이스라엘
구글社 53Q 시제품(19) IBM社 127Q 서비스(22) IonQ社 32Q 서비스(22)	중국과기대 50Q 구축(21)	100Q 개발 (국립양자컴퓨팅센터 ~25년 1,400억원)	50Q 구축 (Micronova, ~24)	100Q 개발 (~30, 700억원)	30~40Q 개발 (국방혁신부, '21~, 700억원)

[ 33]

, 가

[23][24]

가

가 (Mirage)

가

가

가

가

3

2022 5 , 20 20 ( : )  
 110 [25]. (AI) ,  
 5 . ,  
 , 2 [75, 101] [25]. 2022 6  
 [24], “5 , ”  
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 “ ” 5 “ ” 가  
 “ . ( ) ”50  
 가 “ ”.[24]  
 가 .  
 . , 가  
 . , YTN S [26]  
 ( ,  
 ).  
 [ 34] YTN S ,  
 TOP1 (IBM社) IBM  
 IBM 가 /  
 . [ 34] 2 ( ) ,  
 NV  
 , 가 ,  
 , -  
 (Divide-and-conquer Method) .

**백한희 박사, 양자컴퓨터 상용화에 기여**  
**양자컴퓨터 상용화... 한국인 연구업적 공인**  
**한국인 학자 미국 0.5% 석학 됐다**

**백한희** | IBM 원형컴퓨팅 연구원  
 이제 전 세계 모든 사람들이 100 큐비트의 프로세서를 사용할 수 있다는 이야기입니다

**박제근** | 서울대 물리천문학부 교수  
 양자얽힘이라고 하는 것은 초전도 큐비트와 같은 아주 제한된 조건 안에서만 구현된다고 생각했는데 '양자재상양화' 풀었다

**김정삼** | 두크대 전기컴퓨터·인지뇌이공 교수  
 제가 보기엔 기술이 발전될수록 활용 가능한 분야는 무궁무진하게 늘어날 수 있지 않을까

**최가현** 표준과학연구원 박사 (미로 탐색의 경우에) 양자역학을 이용하면 동시에 쫓겨져서 길을 탐색할 수 있어 아무래도

**정연욱** | 삼성전대 양자정보연구지원센터장  
 할 수도 없었던 일을 양자 컴퓨터가 지 열리서 네트워크로 연결돼서

**박희수** | 한국표준과학연구원 양자연구소장  
 양자적일 통신은 우리가 보내고자 하는 메시지를 별도의 암호화과정 없이도 전송할 수 있게 된다

**백정현** | 연세대학교 교수  
 미국 같은 경우에는 차세대 기술인 미국 고유의 드루에 발생하는 양자 얽힘 기술로 산업이 돼서

**한삼록** | KIST 양자정보연구본부장  
 안정적으로 구성되어 있는 탄소 원자들이 들리 떨어져 있기 때문에 모두 변형

**심정현** | 한국표준과학연구원 양자기기이공팀 책임연구원  
 여기 왼쪽에 있는 다이아몬드 센서로부터 발생하는 붉은색 빛의 양이 비광 외부 자기장의 변화와

**유권규** | 한국표준과학연구원 책임연구원  
 (스퀴드 센서는) 이러한 미세 자기장을 측정할 수 있는 센서가 되겠습니다

**양자 센서**  
**스퀴드 센서를 기반으로 한 뇌자도 측정 시스템 개발**

**대형 사고의 발생 방지 가능**

IBM IonQ (美 )  
가 가 .

IBM 100  
. / , .  
YTN .

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2021 11 가 ( “ ” )  
1 , 81.3% ,  
71.8% ( 97.8%, 87.4%, 2019  
) 가 가  
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699 ( , ) . 2021  
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2021 4 . ,  
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(IonQ ) [20]. 11 1  
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前 前 “  
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 /R&D 前 , 20  
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 [33].

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	1. 2. 2021 11 有 3.
	1. - 有 2. 有 - - 가 化
	1. 가 2. 3. - 가 가 有 4. 5. 가 - 有 6. 7. 8.

50

, 2026 50 .  
2026 50 [24].

ETRI KIST 2036 . 가

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[24].

가 [24].

# V.

가

( - ) 가

( ) 가

가

VI

IV 가

가

. 18 가

가

. 20 (Light)

(Photon)

“ ”  $x$   $p(x)$  (Cotangent)

$((x,p) \in T^*X, p = p_i dx^i)$  가

가

( , , )

가

; IBM Qiskit AI

가

가

( ,

). , (Mirage)가  
 ; \_\_\_\_\_( )\_\_\_\_\_  
 \_\_\_\_\_( )\_\_\_\_\_가 , \_\_\_\_\_가  
 \_\_\_\_\_ .

, , ,  
 (Hybrid)  
 - ,

가  
 . 가 IBM 100- 가 ,  
 , IBM 가 ,

(Quantum Supremacy) .  
 IBM , Classical Limit , ( , \_\_\_\_\_  
 \_\_\_\_\_ ) Quantum Limit 가  
 가 .

- \_\_\_\_\_ , \_\_\_\_\_  
 Intel \_\_\_\_\_  
 (Nature) .

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(Python)  
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(Qudit)  
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(National Defence)

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$P_{\text{대한민국 양자기술발전모델}} = M_{\text{양자기술자 그룹의 질량}} V_{\text{양자기술 실적 산출 속도}}$

$M_{\text{양자기술자 그룹의 질량}} \propto N_{\text{양자기술자의 수}} \bar{m}_{\text{양자기술자의 질량}}$

,  $V$  . ,

$V_{\text{양자기술 실적 산출 속도}}$  .

$$\bar{v}_{\text{양자기술 실적 산출 속도}} = \frac{\text{연간 양자기술자 1인의 평균 실적 산출량}}{1\text{년}} < \frac{\text{연간 양자기술자 2인의 평균 실적 산출량}}{1\text{년}} \quad (\text{수학적 상계})$$

;  $M$  .  $M$

$m(m > 0)$  ) ,  $N$  .  $m > 0$

$N$  ; ( ) 1990 2020

267,783 . , 31

8,638 가 ,

가 2 .

, 現 “ ” .

“ ” 가

. , 가 .”

가 ,  $m < 0$  (Training)

$m > 0$  . ,  $m > 0$

“ ” ,

,  $N_{\text{양자기술자의 수}}$  ,

가 가  $N_{\text{양자기술자의수}}$  가 / /  
 (Solution) - \_\_\_\_\_ (Social Physical System)

$P_{\text{대한민국 양자 기술 발전 모델}} m ($   
 $m > 0)$  ( $N_{\text{양자기술자의수}}$ ) . ,  $N_{\text{양자기술자의수}}$   
 $N_{\text{양자기술자의수}}$  /  
 . 20

$N_{\text{양자기술자의수}}$  가 /

가 - ; ,  
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 , “ ”  
 (≒ “ \_\_\_\_\_ 가 ”)

. 가  
 가 .

가 ,  
 \_\_\_\_\_ , 가

\_\_\_\_\_ .  
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 가  
 . (Black Box)  
 - \_\_\_\_\_ (Social  
 Consensus) 가 . ,

가 ;  
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 ( )  
 가 ,  
 가 가  
 - , Quantum Limit 가  
 (Universe) Classical Limit (Earth) ,  
 (Physically Ideal Environment)  
 (Threshold) 가  
 가 가  
 가 (True) , 가 가  
 가 ,  
 (Large-scale)  
 < 10> 가 가  
 가 가 가 “ (Theoretical  
 Quantum Computing-based Quantum Theory; TQCQT)” < 10>;  
 “TQCQT” “ ( ) ”  
 가  
 “TQCQT” ,  
 가 “TQCQT”

$$E_C = \overline{\Psi(x,t)} \hat{H} \Psi(x,t) \in \mathbb{C} \text{ s.t. } \hat{H} \Psi(x,t) = E_C \Psi(x,t) = i\hbar \frac{\partial}{\partial t} \Psi(x,t)$$

$\hat{H} \Psi$

가  $\Psi$   $\hat{H}\Psi$ ,  $\Psi$   
 ( )  
 “ ”  
 $\hat{H}\Psi = \hat{H}(c_1\psi_1 + c_2\psi_2 + c_3\psi_3 + c_4\psi_4 + c_5\psi_5) = c_1\hat{H}\psi_1 + c_2\hat{H}\psi_2 + c_3\hat{H}\psi_3 + c_4\hat{H}\psi_4 + c_5\hat{H}\psi_5$   
 $E_C = \overline{\Psi(x,t)}\hat{H}\Psi(x,t)$  0 (“ ”  
 ; ; 가 (Observable) ) 가  
 (e.g. Hermitian ( ) )  $\hat{H}$   
 가 0 / ).

< 10> “ - 가 ” 가  
 ; , , “TQCQT”

					가	
			1.	2.		
					가	(가)
		$\Psi$			[ 3]	(가)
			$\Psi$	$\Psi$	L=0 가	
			$\Psi$	$\Phi \neq \Psi$	L=1 가	
“ ” “TQCQT”					, 가	
	$\Psi$		1.	2.		
					가	가
		$\Psi$	$\Psi$	$\Psi$		가
		$\Psi$			가	가 ( $\Phi \neq \Psi$ )

가 , “ - 가 ” 가 , ;  
 . “TQCQT” 가 ;  
 가 , < 10> ,  
 “ ( )  
 ‘ , )  
 ” 가 ; “ ”가  
 . ;  
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L=0  
 . ( )  
 ( ) 가 ) 가  
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 , < 10> 가  
 ; 가 (Constrained) (Actual Nature)  
 (Experimental Situation) ;  
 가  
 , 가 가  
 (Theoretical Quantum Computing) (Hybrid)  
 “ ” “TQCQT”, “ ”  
 가 ,  
 , ([ 3]),  
 “ (L=1) (L=0, 1)  
 가 가 , L=0  
 가 . ”  
 ”(= 가 = “ ” ; 가 化 ),  
 ( ) ( ) ;



가 ( ) 가 , ( - ) ( ) 가 ,  
 가 ( 가 ; 가 가  
 가 ) . L=1  
 가 가  
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 가 ;  
 (Ideal) 가 가  
 ( ) .  
 가 , “18  
 가 ” , “가 : ”가 가  
 가 가  
 가

I protest against the use of infinite magnitude as something completed, which in mathematics is never permissible. Infinity is merely a facon de parler(manner of speaking), the real meaning being a limit which certain ratios approach indefinitely near, while others are permitted to increase without restriction.

God does arithmetic. You have no idea, how much poetry there is in the calculation of a table of logarithms!

Mathematics is the queen of sciences and number theory is the queen of mathematics. She often condescends to render service to astronomy and other natural sciences, but in all relations she is entitled to the first rank.

Johann Carl Friedrich Gauss(German: Gauß; 30 April 1777 - 23 February 1855)

■ (Acknowledgement)

가 ,  
 ( , KAIST),  
 UNIST ( 가 ),  
 , Kent State  
 Univ. . GIST ( ),  
 ,  
 NRF  
 ,  
 (NRF , )

■

가  
 ,  
 ( ) ;  
 ( / ) ( ) sschoi@postech.ac.kr  
 (주) (Metacomputing Inc.) ( ) choi@metacomputing.co.kr

※ : 가 NRF ,

# VI.

## 1

[ 1] , (Euler's Formula), (Riemann Zeta Function)

(Axiom)

[1].

-1/12

(Infinite Sum)

가

[ 1]

가

( ) ,

( ) .

### Initial discussion [\[ edit \]](#)

Suppose  $f$  is an analytic function defined on a non-empty open subset  $U$  of the complex plane  $\mathbb{C}$ . If  $V$  is a larger open subset of  $\mathbb{C}$ , containing  $U$ , and  $F$  is an analytic function defined on  $V$  such that

$$F(z) = f(z) \quad \forall z \in U,$$

then  $F$  is called an analytic continuation of  $f$ . In other words, the restriction of  $F$  to  $U$  is the function  $f$  we started with.

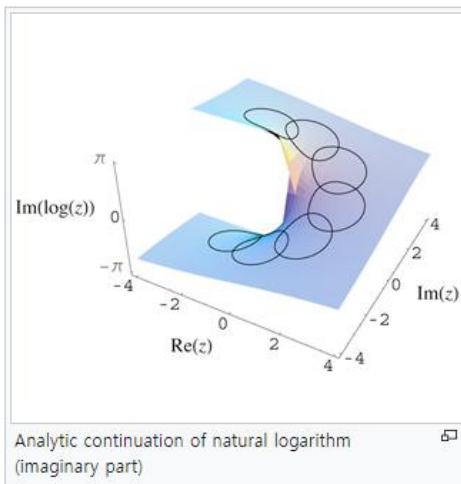
Analytic continuations are unique in the following sense: if  $V$  is the connected domain of two analytic functions  $F_1$  and  $F_2$  such that  $U$  is contained in  $V$  and for all  $z$  in  $U$

$$F_1(z) = F_2(z) = f(z),$$

then

$$F_1 = F_2$$

on all of  $V$ . This is because  $F_1 - F_2$  is an analytic function which vanishes on the open, connected domain  $U$  of  $f$  and hence must vanish on its entire domain. This follows directly from the identity theorem for holomorphic functions.



,  $1, 2, 3, \dots$ ,  $(\eta(s))$

$$s = -1$$

; 가 가 )

$$1 + 2 + 3 + \dots = \sum_{n=1}^{\infty} \frac{1}{n^{-1+i0}} = \left( \sum_{n=1}^{\infty} \frac{1}{n^s} \right)_{s=\sigma+it=-1+0\tau}$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad (Re(s) > 1) = \sum_{n=1}^{\infty} \left( \frac{1}{(2n-1)^s} + \frac{1}{(2n)^s} \right)$$

$$\Rightarrow - \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} + \sum_{n=1}^{\infty} \frac{1}{n^s} = 2 \sum_{n=1}^{\infty} \frac{1}{(2n)^s} = 2^{1-s} \sum_{n=1}^{\infty} \frac{1}{n^s} = 2^{1-s} \zeta(s)$$

$$\Rightarrow -\eta(s) + \zeta(s) = 2^{1-s} \zeta(s) \Rightarrow -\eta(s) = \zeta(s)(2^{1-s} - 1)$$

$$\Rightarrow \zeta(s) = \frac{\eta(s)}{1 - 2^{1-s}} \quad (Re(s) \neq 1) \Rightarrow \zeta(-1+i0) = \frac{1/4}{1-4} = -\frac{1}{12}$$

$$\therefore 1 + 2 + 3 + \dots = \left( \sum_{n=1}^{\infty} \frac{1}{n^s} \right)_{s=-1+i0} \quad (Re(s) = -1 \neq 1)$$

$$\text{해석적 연속} \Rightarrow \frac{\eta(s)}{1 - 2^{1-(-1+i0)}} = \zeta(-1+i0) = -\frac{1}{12} \quad (\text{해석적 연속의 결과는 유일함})$$

가  
가

( )

가 0

가

가

, (Quantum Electrodynamics; QED) “ ”  
 , (Renormalization) . ,  
 “ ”  
 “ ”  
 ; . , 가  
 “ ” 가

2

( : Complex Number Field) (Geometric Intuition)  
 가 (Algebraic Number Field)  
 (Fields Medal)  
 (The Langlands Programme) (※  
 ). (Bottom-up)  
 가  
 , (Fundamental Lemma) 2010  
 (Ngô Bảo Châu, June 28, 1972 ~) ,  
 (Automorphic Form) (String Theory) (Brane)  
 ; (Topological Quantum Field  
 Theory; TQFT) (Supersymmetry Theory) , -  
 (Kapustin-Witten) (Twist) (Electromagnetic Duality)  
 (Geometric Langlands Correspondence) (Langlands Reciprocity)



■                    가 (Langlands Conjecture)

갈루아 군(Galois Group)의 모든  $n$  차원적 기약표현(Irreducible Representation(Rep.))  
 $\rho: G(\bar{K}/K) \rightarrow GL_n(\mathbb{C})$ 에 대해 다음을 만족하는 보형표현(Automorphic Rep.)  $\pi$ 가 존재한다:  
 $L(s, \rho) = L(s, \pi)$  & 거의 모든 Frobenius 고유값들이 Hecke 연산자 고유값들에 대응(correspond to).  
 Artin  $L$ -function:  $L(s, \rho) = \prod_{v < \infty} \left( \det \left( 1 - \rho(\text{Frob}_v) (N(v))^{-s} \right) \right)^{-1}$   
 ( $s$ : 복소변수,  $\text{Frob}_v$ : Frobenius 자기동형사상(Automorphism),  $N(v)$ : 이데알  $v$ 의 Absolute Norm)

가 (Riemann Hypothesis)  
 가 (Generalized Riemann Hypothesis)                    가 (Grand Riemann Hypothesis)  
 $L$ - (Artin  $L$ -function)

■                    가 (Grand Riemann Hypothesis)

$L(s, \chi)$	(Nontrivial Zero)	(Real Part)	1/2
--------------	-------------------	-------------	-----

$\chi$  (Dirichlet Character) . , 가  
 가 (Automorphic Information)  
 (Arithmetic Information) . (Prime Numbers;  
 Primes) (Eigenvalues) .  
 (Reduction to the Absurd) (Contradiction)  
 $f(q)$ 가 가

■ (Fermat's Last Theorem)

가정:  $u^p + v^p = w^p$  ( $p \geq 3$ ) 을 만족하는  $u, v, w \in \mathbb{Q}$  ( $uvw \neq 0$ ) 존재  
 즉, 타원곡선  $E: y^2 = x(x+u^p)(x-v^p)$  존재

Normalized Newform for  $E: \exists f(q) \in S_2(\Gamma_0(N_E))$  with  $N_E = \prod_{l|uvw} l$

(Wiles, Taylor-Wiles)  $\left( \text{거의 모든 소수 } p \text{ 에 대해 } a_p = \text{Tr} \left( \rho \left( \text{Frob}_p^{\text{geom}} \right) \right) \text{ 인} \right)$

- $\ast \rho: G_{\mathbb{Q}} \rightarrow \text{GL} \left( H_{\text{et}}^1 \left( E_{\overline{\mathbb{Q}}}, \overline{\mathbb{Q}}_l \right) \right) \cong \text{GL}_2 \left( \overline{\mathbb{Q}}_l \right)$  (étale comparison)
- $\ast S_2(\Gamma_0(N))$ :  $\Gamma_0(N)$  위의 모든 Cusp Forms (of weight 2)
- $\ast \Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}_2(\mathbb{Z}) : c \equiv 0 \pmod{N} \right\}$

Normalized Newform (Mazur, Ribet) :  $\exists g \in S_2(\Gamma_0(2))$   
 (어떤 소수와  $f, g$  푸리에 계수들과 합동인)

모순:  $S_2(\Gamma_0(2)) = \{0\} \Rightarrow g$  존재 불가  $\Rightarrow f$  존재 불가  $\Rightarrow$  가정에 모순

결론:  $u^p + v^p = w^p$  ( $p \geq 3$ ) 을 만족하는  $u, v, w \in \mathbb{Q}$  ( $uvw \neq 0$ )는 존재하지 않음

(Étale Comparison  
Theorem)  
(Alexander Grothendieck, 28 March  
1928 - 13 November 2014)  
(Étale Cohomology)

가  
(Algebraic Geometry)

(Automorphic Form) (Langlands  
Correspondence) (Number)

가  
가 (Discrete) (Algebraic Number)가 (Continuous)  
(Geometric)

(Microscopic Physics)

(String Theory) , (Quantized) .  
 (Automorphic Form) , (Modular Form)

**Enter number theory in physics**

We package the microstates into a **partition function**:

$$Z(\beta) = \sum_{\text{states}} e^{-\beta E_{\text{states}}} \quad \beta = 1/T$$

In string theory the partition function is an **automorphic form!**

Finding the black hole microstate degeneracies corresponds to calculating **Fourier coefficients** of automorphic forms

[ 35] , [6]

2019 Nature Reviews Physics “ (Strings and Qubits)” ,  
 (Quantum Information) [4]. ,  
 / (Gauge/Gravity Correspondence)  
 ( 가 가 ).  
 /M- [5].  
 (Fundamental Lemma) ,  
 4 - (Yang-Mills) (Hitchin System)  
 (Étale Cohomology) . -  
 (Quantum Field Theory) (Strong Force) (Weak Force)  
 . (Quantum Computer) .  
 4 - (Classical Mechanics)  
 ; 가 (Meromorphic )  
 . Meromorphic Generalization ,

(Analytic Continuation)

(Continuous) (Discrete)  
 (Algebraic Number) ,  
 (Algebraic Geometry) .  
 (Automorphic Form) ( , )가 ,

3

[ 1]  $m$  가  $n$   
 (Real Vector Space) ,  
 $V: \mathbb{R}^n \rightarrow \mathbb{R}$   $V$  가 ,  
 $t$   $x(t) \in \mathbb{R}^n$  ,  $p(t) \in \mathbb{R}^n$   $x(t)$   
 (Cotangent)  $((x,p) \in T^*X, p = p_i dx^i; x, p)$  ;  
 (Newtonian Physics)  $v(t) = dx(t)/dt, p(t) = mv(t)$  .  
 $(x(t), p(t))$   $H: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$   $H(x,p) := \frac{|p|^2}{2m} + V(x)$   
 $m$   $(x(t), p(t))$  (Energy  
 Conservation)  $dH(x,p)/dt = 0$  ,  
 (Time-independent)  $E$  ,  $H(x,p) = E$  .

[ 2]  $\psi(x,t)$   $\psi: \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{C} (|\psi|^2 := 1)$  .  
 $t$  가 , 가  
 가 (Normalizable) (

$\langle \phi, \psi \rangle := \int_{\mathbb{R}^n} \overline{\phi(x,t)} \psi(x,t) dx, |\psi|^2 := \langle \psi, \psi \rangle = 1$  if  $\psi$  is normalizable (내적에 의한).  
 $\langle a(t) \rangle := \langle \psi(x,t), \hat{A} \psi(x,t) \rangle, \langle a(t) \rangle$ : 기댓값 if  $\psi$  is normalizable (내적에 의한).

내적에 의한 정규화( $\langle \psi, \psi \rangle = 1$ ) 불가時  $|\psi|^2 := \bar{\psi}\psi = 1$ 이 되도록 정규화,  
 이 때, 기댓값은  $\langle a(t) \rangle := a(t)$  where  $\hat{A}\psi(x,t) = a(t)\psi(x,t)$  와 같이 정의.

(Hilbert Space)

(Inner Product)

$$\langle x(t) | p(t) \rangle = \int_{\mathbb{R}^n} \overline{\psi(x,t)} \hat{X}\psi(x,t) dx = \int_{\mathbb{R}^n} \overline{\psi(x,t)} x \psi(x,t) dx \Rightarrow \hat{X}\psi(x,t) = x\psi(x,t)$$

$$\langle p(t) | x(t) \rangle = \int_{\mathbb{R}^n} \overline{\psi(x,t)} \hat{P}\psi(x,t) dx = \int_{\mathbb{R}^n} \overline{\psi(x,t)} p \psi(x,t) dx \Rightarrow \hat{P}\psi(x,t) = p\psi(x,t)$$

(\* Self-adjoint  $\Leftrightarrow$  0).

※ : Page 26  $\Rightarrow$  \_\_\_\_\_ / \_\_\_\_\_

⟨Analogy in General Relativity (vs. Classical Mechanics)⟩

Find Complex Functions  $\hat{G}_{\mu\nu}, \hat{g}_{\mu\nu}, \hat{T}_{\mu\nu}$  for

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \quad (\text{아인슈타인 장 방정식: EFE})$$

$$\psi_{\mu\nu}(x) \approx c_{\mu\nu} e^{-ik_\alpha x^\alpha}, \quad x^\alpha = (ct, x, y, z), \quad k_\alpha = \left( \frac{\omega}{c}, -k_x, -k_y, -k_z \right)$$

$$\langle \phi, \psi \rangle := \int \bar{\phi} \psi, \quad \langle \psi, \hat{A}\psi \rangle := \int \bar{\psi} \hat{A}\psi \quad (\text{Curved Space-time 적분})$$

Normalization Condition:  $|\psi|^2 := \bar{\psi}\psi = 1$

"Expectation Value":  $\langle A_{\mu\nu}(x) \rangle := A_{\mu\nu}(x)$  where  $\hat{A}_{\mu\nu}\psi_{\mu\nu}(x) = A_{\mu\nu}(x)\psi_{\mu\nu}(x)$

Inner Product-like Normalization Condition:  $|\psi|^2 := \langle \psi, \psi \rangle = 1$

"Expectation Value":  $\langle A_{\mu\nu}(x) \rangle := \langle \psi_{\mu\nu}(x), \hat{A}_{\mu\nu}\psi_{\mu\nu}(x) \rangle$

$$\text{※ } -ik_\alpha x^\alpha = i(\mathbf{k} \cdot \mathbf{x} - \omega t) = i \left( k_x x + k_y y + k_z z - ct \sqrt{k_x^2 + k_y^2 + k_z^2 + \left( \frac{m_0 c}{\hbar} \right)^2} \right)$$

$m_0$ : 정지 질량, **Problem: 비선형 (Nonlinear)**

General Relativity	Newtonian Physics
$T_{\mu\nu}(x^\alpha) \rightarrow \langle T_{\mu\nu}(x^\alpha) \rangle$	$E(t) \rightarrow \langle E(t) \rangle$
$\langle T_{\mu\nu}(x^\alpha) \rangle := \int_{\partial_\alpha \Omega} \overline{\psi_{\mu\nu}(x)} \hat{T}_{\mu\nu} \psi_{\mu\nu}(x) dS^\alpha$	$\langle E(t) \rangle := \int_{\mathbb{R}^n} \overline{\psi(x,t)} \hat{E} \psi(x,t) dx$
$\hat{T}_{\mu\nu} \psi_{\mu\nu}(x) = T_{\mu\nu} \psi_{\mu\nu}(x)$	$\hat{E} \psi(x,t) = E \psi(x,t)$
$\partial_\nu T^{\mu\nu} = 0$	$\partial_t E = 0$
$T_{\mu\nu} = G_{\mu\nu} = \frac{c^4}{8\pi G} \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} \right) = \mathcal{L}_M g_{\mu\nu} - 2 \frac{\delta \mathcal{L}_M}{\delta g_{\mu\nu}}$	$E = H = \frac{ p ^2}{2m} + V$

\* : Analogy

(  $\hat{A}$ : Self-adjoint )  
 $\langle a(t) \rangle = \langle \psi(x,t), \hat{A} \psi(x,t) \rangle = \int_{\mathbb{R}^n} \overline{\psi(x,t)} \hat{A} \psi(x,t) dx = \int_{\mathbb{R}^n} \hat{A} |\psi(x,t)|^2 dx = \int_{\mathbb{R}^n} a |\psi(x,t)|^2 dx$   
 즉, 통상  $\langle a(t) \rangle = \langle \psi(x,t), \hat{A} \psi(x,t) \rangle = \langle \hat{A} \psi(x,t), \psi(x,t) \rangle$  와 같은 경우들을 다루려 함을 유의.  
 (\* Self-adjoint Hermitian )

[ 3 ] ( )

$\psi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{C}$

( $\leftrightarrow$ )

가 ; “ ”

“ ”

$\hat{P} : \mathbb{C} \rightarrow \mathbb{C}$

$\hat{P}(\psi(x,t))$  ( $\hat{P} \circ \psi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{C}$ )

가

$\hat{P} : \mathbb{C} \rightarrow \mathbb{C}$

$\hat{P}$

(

- ) ; 가 \_\_\_\_\_ 2 가

(e.g.

가 ). ,  $\hat{P}$   $\hat{P}(\psi(x,t))$ 가

(Boundary Condition)

$\hat{P}$

가

$\hat{P}$

가 , 가 . 가 가  
 (Uniqueness Theorem)  
 (Reduction to the Absurd)

[ 4 ]

$$\langle E(t) \rangle = \langle \psi(x,t), \hat{E} \psi(x,t) \rangle = \langle \psi(x,t), E \psi(x,t) \rangle \Rightarrow \hat{E} \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t) = E \psi(x,t) = H \psi(x,t) = \hat{H} \psi(x,t)$$

$$\langle H(t) \rangle = \langle \psi(x,t), \hat{H} \psi(x,t) \rangle = \langle \psi(x,t), H \psi(x,t) \rangle \Rightarrow \hat{H} \psi(x,t) = H \psi(x,t) = E \psi(x,t) = \hat{E} \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t)$$

$$\begin{aligned} \langle H(t) \rangle &= \langle \psi(x,t), H \psi(x,t) \rangle = \int_{\mathbb{R}^n} \overline{\psi(x,t)} \left( \frac{|p|^2}{2m} + V(x,t) \right) \psi(x,t) dx \\ &= \left\langle \psi(x,t), \frac{|p|^2}{2m} \psi(x,t) \right\rangle + \langle \psi(x,t), V(x,t) \psi(x,t) \rangle \\ &= \left\langle \psi(x,t), \frac{1}{2m} \hat{P} \hat{P} \psi(x,t) \right\rangle + \langle \psi(x,t), \hat{V} \psi(x,t) \rangle \\ &= \langle \psi(x,t), \hat{H} \psi(x,t) \rangle \end{aligned}$$

↓

$$\begin{aligned} \left\langle \psi(x,t), \frac{1}{2m} \hat{P} \hat{P} \psi(x,t) \right\rangle &= \left\langle \psi(x,t), -\frac{\hbar^2}{2m} \nabla_x^2 \psi(x,t) \right\rangle = \frac{1}{2m} \left\langle \psi(x,t), \frac{|p|^2}{2m} \psi(x,t) \right\rangle \\ &\Rightarrow \frac{1}{2m} \hat{P} \hat{P} \psi(x,t) = -\frac{\hbar^2}{2m} \nabla_x^2 \psi(x,t) = \frac{|p|^2}{2m} \psi(x,t) \end{aligned}$$

$$\begin{aligned} \langle \psi(x,t), \hat{V} \psi(x,t) \rangle &= \langle \psi(x,t), V(x,t) \psi(x,t) \rangle \\ &\Rightarrow V \psi(x,t) = V(x,t) \psi(x,t) \end{aligned}$$

$$i\hbar \frac{\partial}{\partial t} = \hat{E} = \hat{H} = -\frac{\hbar^2}{2m} \nabla_x^2 + \hat{V} = -\frac{\hbar^2}{2m} \nabla_x^2 + V(x,t) \Rightarrow i\hbar \frac{\partial}{\partial t} \psi(x,t) = E \psi(x,t) = \left( -\frac{\hbar^2}{2m} \nabla_x^2 + V(x,t) \right) \psi(x,t)$$

$$\hat{\Xi}: \mathbb{C} \rightarrow \mathbb{C}, \quad \hat{\Xi} \psi(x,t) := \left( i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla_x^2 - V(x,t) \right) \psi(x,t)$$

슈뢰딩거 방정식 : 함수  $s = \hat{\Xi} \psi(x,t)$ 의 영점 ( $s = 0$ )

[ 5 ]

( )

“ ” ; ,

$$( F = ma ) ;$$

Quantum Limit    Classical Limit    ( : ).

**Quantum theory of small amplitude lattice vibrations: Second quantization of LHO**

→ Second Quantization applied to system of Linear Harmonic Oscillators:

$$\hat{Q}_i(\mathbf{q}) \hat{P}_i(\mathbf{q}) \rightarrow \hat{a}_i(\mathbf{q}), \hat{a}_i^\dagger(\mathbf{q})$$

$$\hat{Q}_i(\mathbf{q}) = \sqrt{\frac{\hbar}{2\omega_i(\mathbf{q})}} (\hat{a}_i(\mathbf{q}) + \hat{a}_i^\dagger(-\mathbf{q}))$$

$$\hat{P}_i(\mathbf{q}) = -i\sqrt{\frac{\hbar\omega_i(\mathbf{q})}{2}} (\hat{a}_i(\mathbf{q}) - \hat{a}_i^\dagger(-\mathbf{q}))$$

$$\Rightarrow \hat{H} = \sum_{\mathbf{q}} \hbar\omega_i(\mathbf{q}) \left( \hat{a}_i^\dagger(\mathbf{q}) \hat{a}_i(\mathbf{q}) + \frac{1}{2} \right)$$

canonical transformation  $[\hat{a}_i(\mathbf{k}), \hat{a}_i^\dagger(\mathbf{q})] = \delta_{\mathbf{k},-\mathbf{q}}$ ,  $[\hat{a}_i(\mathbf{k}), \hat{a}_i(\mathbf{q})] = [\hat{a}_i^\dagger(\mathbf{k}), \hat{a}_i^\dagger(\mathbf{q})] = 0$

→ Hamiltonian is a sum of  $3N$  independent LHO – each of which is a referred to as a **phonon mode!** The number of phonons in state  $(\mathbf{q}, i)$  is described by an operator:

$$\hat{n}_i(\mathbf{q}) = \hat{a}_i^\dagger(\mathbf{q}) \hat{a}_i(\mathbf{q})$$

PHYS 624 Crystal Lattice Vibrations: Phonons

Temperature dependence of the mean energy  $\langle E \rangle$  of a quantum harmonic oscillator:

$$\langle E \rangle = \frac{1}{2} \hbar\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$$

**Low Temperature Limit:**  $\hbar\omega \gg k_B T$

**“Zero Point Energy”**

At low T, the exponential in the denominator of the 2nd term gets larger as T gets smaller. At small enough T, neglect 1 in the denominator. Then, the 2nd term is  $e^{-x}$ ,  $x = (\hbar\omega/k_B T)$ . At very small T,  $e^{-x} \rightarrow 0$ . So, **in this case,  $\langle E \rangle$  is independent of T:**  $\langle E \rangle \rightarrow (\frac{1}{2})\hbar\omega$

Temperature dependence of the mean energy  $\langle E \rangle$  of a quantum harmonic oscillator.

Taylor's series expansion of  $e^{-x} \rightarrow \phi^x = 1 + x + \frac{x^2}{2!} + \dots$  for  $x \ll 1$

$$\langle E \rangle = \frac{1}{2} \hbar\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$$

**High Temperature Limit:**  $\hbar\omega \ll k_B T$

At high T,  $\langle E \rangle$  is independent of  $\omega$ . This high T limit is equivalent to the classical limit, (the energy steps are small compared to the total energy). So, **in this case,  $\langle E \rangle$  is the thermal energy of the classical 1D harmonic oscillator** (given by the equipartition theorem).

$\langle E \rangle \approx \frac{1}{2} \hbar\omega + k_B T$

[ 36 ] : Limit vs. Limit [34][35]

[ 6 ] , 가 ( ) .  
 ( ) 가 0  
 $\psi(x, t)$  (3 ,  $\psi : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{C}$ )  
 $\hat{P} : \mathbb{C} \rightarrow \mathbb{C}$  “  
 ” 가 0 ” , 가 0  
 가 - \_\_\_\_\_ ;  
 1 1

[ 7 ] EPR , “ ” “ ”  
 “ 가 ”  
 ( \_\_\_\_\_ )  
 가 (※ \_\_\_\_\_ )

[ 8 ] \_\_\_\_\_ , \_\_\_\_\_  
 ( ) “ ”  
 $\hat{X}, \hat{P}$  ( ) ( - )

		1. ( )	2. ( )
1	$\Psi(r, \theta, \phi, 1)$	$\langle \Psi(r, \theta, \phi, 1), \hat{X} \Psi(r, \theta, \phi, 1) \rangle$	$\langle \Psi(r, \theta, \phi, 1), \hat{P} \Psi(r, \theta, \phi, 1) \rangle$
2	$\Psi(r, \theta, \phi, 2)$	$\langle \Psi(r, \theta, \phi, 2), \hat{X} \Psi(r, \theta, \phi, 2) \rangle$	$\langle \Psi(r, \theta, \phi, 2), \hat{P} \Psi(r, \theta, \phi, 2) \rangle$
3	$\Psi(r, \theta, \phi, 3)$	$\langle \Psi(r, \theta, \phi, 3), \hat{X} \Psi(r, \theta, \phi, 3) \rangle$	$\langle \Psi(r, \theta, \phi, 3), \hat{P} \Psi(r, \theta, \phi, 3) \rangle$

가 ; 가 가

(Hidden Variable) , “ 가 ” L .

L			1. ( )	$\Psi$	2. ( )
0	×		$\langle \Psi, \hat{X}\Psi \rangle$	$\Leftarrow \Psi$	$\Rightarrow \langle \Psi, \hat{P}\Psi \rangle$ $\Phi = \hat{X}\Psi \Rightarrow \langle \Phi, \hat{P}\Phi \rangle$
1	○	_____	$\langle \Psi, \hat{X}\Psi \rangle$	$\Leftarrow \Psi$	$\Phi = \hat{X}\Psi \Rightarrow \langle \Phi, \hat{P}\Phi \rangle$

L=1 ,  $\langle \Psi, \hat{X}\Psi \rangle$   $\langle \Psi, \hat{P}\Psi \rangle$  ;  
 $\Psi(r, \theta, \phi, 1)$   $\langle \Psi, \hat{X}\Psi \rangle$  ,  $\langle \Psi, \hat{P}\Psi \rangle$   
 $\Psi(r, \theta, \phi, 1)$   $\langle \Psi, \hat{X}\Psi \rangle$  , L=1  
 $\Psi(r, \theta, \phi, 2)$ 가 가 ( L=0 L=1 ( × ) .

L=0: “  $[\hat{X}, \hat{P}] = i\hbar$  ×  $\Rightarrow ([\hat{X}, \hat{P}] = i\hbar)$  ”,  
 L=1: “  $[\hat{X}, \hat{P}] = i\hbar$  ○  $\Rightarrow ([\hat{X}, \hat{P}] = i\hbar)$  ”.

[ 9] (※ . )

L=1	_____ .	
1	_____ ( ) 가 有	_____ ( )
2	. : 0 0 “ ”	$\hat{H} = \hat{H}^\dagger$ $\hat{H}$

$(r, \theta, \phi, t)$   
 $(\quad)$   
 $(\quad) \Psi(r, \theta, \phi, t)$   
 $L=0$  “ ”  $\hat{X}\Psi(r, \theta, \phi, t)$   $\hat{P}\Psi(r, \theta, \phi, t)$   
 . [ 8] ,  $L=0, 1$  가 ,  
 $(\quad)$  가 )  $L=1$  가 .  


---

 $L=0$   $L=1$  가 ; ,  $L=0$   $L=1$   
 .  $(L=1)$  (Image)  
 $(Image)$  가  $L=1$   
 $(L=0)$   $(Image)$   $L=1$  ). (\*  $\frac{\quad}{\quad}$ )  


---

 , 가 가)  
  
 [ 10]  $L=0, 1$  가 ,  
 .  $F = ma$   $L=0$  ,  
 $L=1$  .  
 $L=0$   $(\quad)$  .  
  
 [ 11] “ $(\quad, \quad, \quad)$  ” ,  $L=0$   
 ,  $L=0$   
 . ,  $(L=1)$   
 “ ”  $(\quad)$  .  $L=0$   
 ,  $L=0$   
 . 가 ,  
 (\*  
 ).  $L=0$   $L=1$  가  


---

 $L=0$   $L=1$  가 가 (\*  $\rightarrow$  , ).

# 4

[ 1] (Complex Number Field) (Product Operator)가 (Nature) 가

$\psi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{C}$ , (Rotation) (Product) (Operation) (Algebraically Closed) .  $\mathbb{C} \mathbb{R}$  ,

;  $\psi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{C}$  (Nonzero Eigenvector) 가 , (Non-trivial Solution) 가

$T$  (Matrix Representation)  $A$  ,  $A$   $\lambda$ (Eigenvalue) (Monomial Polynomial)  $p(\lambda) = \det(A - \lambda I) = 0$  (Fundamental Theorem of Algebra) ,  $p(\lambda) = 0$  (Complex Root) 가 ;  $A$ 가 “ ” 가

가 “ 가? ”

;  $\mathbb{N}$  가 (e.g.  $\mathbb{N}$  3 7 3-7=-4 , -4  $\mathbb{N}$  가 ). ,  $\mathbb{N}$  ,  $\mathbb{N}$  가 “ ” 0  $\mathbb{N}$   $\mathbb{Z}$ (Ring)  $\ast \mathbb{N}$  ) ;  $\mathbb{R}$  가 “ ”  $\mathbb{C}$  .

(Diverging to Infinity) / , 가 ( , “ ” 가 ; 1  $1 := 1 + i\tau$  ( $\tau = 0$ ) .

$s = 1 + i0$  (Radius)  $r$  (Circle) 가 . ,  
 1 1 .  
 ). N 가 “ ” ,  
 가 R “ ”  
 .  
 , 가 “C R  
 ” 가 , “Z N  
 ”  
 가 . , 가  
 Z( ) , C  
 가 .  
 , 가  $\psi : \mathbb{R}^n \rightarrow \mathbb{R}$  ( ,  
 )  
 , (Hermitian Operator)  $\psi$ 가  
 가 ; , 가 ,  
 가 .  
 가 .  
 , “ C (A Useful Mathematical  
 Tool) ” ( 가  
 ), : “ (Digonalization)가  
 가 ” . ,  
 “ ” ,  $\psi : \mathbb{R}^n \rightarrow \mathbb{C}$   
 .  
 [ 2] 가 “ ”  $\mathbb{R}^n \rightarrow \mathbb{R}$   
 가 ,  $\mathbb{R}^n \rightarrow \mathbb{C}$   
 “ ” . , (Real Analysis) ,  
 (Complex Analysis) 가 . ,

가

(Georg Friedrich Bernhard Riemann, 17 September 1826 – 20 July

1866) (가). 가  
가 ( )

(David Hilbert, January 1862

~ February 1943) “ 가  
” 가 (Hilbert Space)  
1900 “

23가 ”  
가 , (Axiomatization) 가  
(23가 )

(同)

(Riemannian Geometry)

[ 3]

( 가 ;  
) 가 t  
(Localized) ; (Non-localized)

(Expectation Value)

; 가 0

(Basis)

가

[ 4]

가  $\mathbb{R}^n \rightarrow \mathbb{C}$   $\mathbb{R}^n \rightarrow \mathbb{R}$ ,  
(Algebraically Closed)

“ ( ) “ 가 0  
 ”( “ ”)  
 $\psi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{C}$   $\psi : \mathbb{R}^n \rightarrow \mathbb{C}$ 가 (Hermitian  
 Operator) (Hamiltonian)  
 (Modelling) . ,  
 , 가 0 ( ), ( )  
 가 ).

## 5 가

[ 1]  $\psi(r, \theta, \phi, t)$   $[\hat{L} \cdot \hat{S}, \hat{L}_z] \neq 0$  가  
 $\psi$   
 가

$$\sum_{m_s} C\left(l, \frac{1}{2}, j; m_j - m_s, m_s, m_j\right) Y_l^{m_j} \chi_{m_s} \Rightarrow \begin{pmatrix} \sqrt{\frac{\pm k + \frac{1}{2} - m}{2k + 1}} Y_{\pm k}^{m - \frac{1}{2}}(\theta, \phi) \\ -\text{sgn}(k) \sqrt{\frac{\pm k + \frac{1}{2} + m}{\pm 2k + 1}} Y_{\pm k}^{m + \frac{1}{2}}(\theta, \phi) \end{pmatrix}$$

$C$ : Clebsch-Gordan Coefficients,  $\chi$ : Spinor

– (Clebsch-Gordan coefficient) (Tensor Product;  
 $\otimes$ ) (Irreducible Representation) (Direct Sum;  $\oplus$ )

, . 0  
 , 가 ,  
 가 (Commutative Group) 가

$\psi$  (Relation) . 가  
 $\psi$   $Y, \chi$   $\otimes$   
 $\hat{L} \cdot \hat{S}$   $\hat{L}_z$

. (  $\hat{H}$  )  
 ) ; 가 가 ,

가  
 가 .  
 가  
 ( , 가 가  
 ; ).

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(Superposition)  
 (Quantum Superposition) ,  
 (  $\sum$  )  

$$\Psi = \sum_{\psi} c_{\psi} \psi$$
 ( , 가  
 ( 가  
 - Tensor Networks (Data-sparse Approximation)  
 (Approximate) ).

[ 3] (Quantum Computing)

(Parallel Computing) , , 
$$\Psi = \sum_{\psi} c_{\psi} \psi$$
  
 ( , 가  
 ).  $\Psi$  , 가  
 ,  $c_{\psi} \psi$   
 .  $\Psi$  가 , 가  $\Psi$   
 ( ) .  

$$\hat{O} \Psi = \sum_{\psi} c_{\psi} \hat{O} \psi$$



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